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Capacity expansion with technological change

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TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER 1. INTRODUCTION

1.1 Overview

Capacity expansion is the process of adding new facilities of similar types over time to meet a rising demand for their services. Planning for the expansion of capacity is of vital importance in many applications within the private and public sectors. Examples can be found in heavy process industries, communication networks, electrical power service, and water resource systems. Capacity expansion planning consists of determining future expansion times, sizes, locations and types of facilities in the face of uncertain demand forecasts, costs, and completion times.

Technological progress is an important factor to be considered in capacity expansion problems. Traditionally, improvements in technology are measured either in terms of increased revenue associated with the new technology or decreased costs of procurement and operation of the new technology. In many industries, such as commercial satellite communications or computer central processing units, introduction of the new improved technology causes increases in product efficiency and rapid decreases in unit costs, thus affecting the expansion planning decision. Technological change also enlarges markets indirectly through improved productivity. Productivity improvements reduce production costs. Falling costs enable price reductions and expand the customer base and thus the market.

The effect of a construction lead time for adding new capacity is also an important issue in capacity expansion problems. If a lead time for adding capacity exists, the capacity expansion problem is more complicated because uncertain demand creates the risk of shortage during the construction period, which can be very costly. If there are no lead times for adding new capacity, despite the uncertainty of demand there would be no risk of capacity shortage, since the manager could simply wait until demand equals current capacity and then install new capacity.

1.2 Statement of problem

In many new technology industries, demand for capacity grows according to an exponential trend. For example, recent work by Dumortier (1997) predicted the exponential growth in the number of Internet users and the end-user multimedia applications. Rai et al. (1998) used the number of service hosts as the measure of the Internet size and suggested that exponential model provided the closest fit with the increasing number of service hosts. Kruger (2000) predicted the significant growth of electric consumption due to substitution of hydrogen for fossil fuels in motor vehicles. The major concern is the magnitude of additional electricity power capacity necessary to build a large-scale hydrogen fuel industry, especially in a state with large number of vehicles such as California.

Technological progress is an important consideration for capacity expansion. New technological improvements motivate a competitive market and increase company assets. Several examples show how technological progress could influence the cost of expansion. In Snow's observation (1975) the per-unit capacity cost of satellite communication INTELSAT was decreased significantly due solely to technological progress. Newer, improved technology of the satellite component expanded voice channel capacity. During the 12 years of consideration, the voice channel capacity had been increased from 480 to 25000, which is more than 50 times, while the capital cost per satellite increased only 3.77 times from 16.5 to 62.3 million dollars. This ratio of channel increase to capital cost increase clearly shows how technological improvement could affect the cost of expansion. Moore's Law, which stated that computer CPU speed would be doubled up once very 18 months, is another distinctive example of how technological progress impact the cost of expansion in many industries. The existence of Moore's Law creates improvement in technology and enhances production ability, while causes older technology to become obsolete and prices to drop regularly. With this technological progress, a manager can choose to purchase the latest technology at the highest price, or purchase the older technology for a lower price.

With all the examples mentioned above, we can see that a capacity expansion problem with exponential demand growth could be more complicated under the technological progress environment. The total cost of expansion over a long horizon will be considerably different from the expansion problem with stationary technology. Some research in the past explored various capacity expansion problems and determined the optimal policies for those cases, but none of them have combined the consideration of random exponential demand growth and the uncertain technological change together. This research concerns both combining those considerations and investigating the optimal capacity expansion policies.

1.3 Research objective

In this thesis, we explore the capacity expansion problem in several model scenarios. Each of them will be formulated according to the types of demand growth, the type of technological change and the existence of a lead time of capacity expansion. Types of demand growth include deterministic exponential and random exponential, while types of technological change include deterministic progress and uncertain progress. The goal of this thesis is to determine the impact of technological change on the optimal timing and sizes of capacity expansions to minimize the expected expansion cost while controlling any risk of shortage.

We formulate a dynamic programming model of capacity expansion for each problem case. The objective function for the optimal policy is the total cost of expansion over an infinite time horizon. In the deterministic demand growth problem and the random demand growth without lead time problem, the objective function consists of the expected discounted cost of capacity expansion. In these cases, expansions are made when demand reaches current capacity, so timing and sizing decisions are not separate. In the other problem case with the existence of a lead time of expansion, the cost of shortage will be combined into the objective function. In this case, we determine the optimal expansion policy in two dimensions so that both future demands are satisfied to an acceptable level and the expansion costs are minimized. The first dimension concerns timing as we mentioned above. The second dimension concerns size, which is determining how much capacity to be added at given time in the view of discounted cost and economies of scale. With this form of policy, we can solve for the optimal policy parameters to minimized a weighted combination of total discounted expected expansion cost and the cost of shortage.

The most important benefit of this research is the ability to obtain an optimal policy for the capacity expansion problem in the face of exponential demand growth and the existence of technological progress. This research is especially relevant in the service industries, which often are faced with intensely competitive markets and wish to avoid the potential risk of shortage.

1.4 Thesis organization

This thesis consists of six chapters. The first chapter describes the introduction, objectives, and scope of this thesis. Chapter 2 reviews the past literature relevant to capacity expansion, technological progress and its impact on the total cost of expansion, lead time of construction, and the use of financial option pricing to estimate the shortage. Notations for the parameters used in this thesis are also included in this chapter.

From Chapter 3 to Chapter 5, we discuss the capacity expansion problem in several cases defined by the type of demand growth, the nature of technological progress and the existence of a lead time lead time expansion. Figure 1.1 illustrates the organization of this thesis. In each chapter, we formulate the capacity expansion model and perform the calculations to investigate the effect of technological progress on the cost of expansion over an infinite time horizon. In Chapter 3, we investigate the model with deterministic demand growth. In Chapter 4, we discuss the model with random demand growth. Finally, in Chapter 5, we add the consideration of lead time of expansion into the capacity expansion model with random demand growth. General concluding remarks and future works are described in Chapter 6.

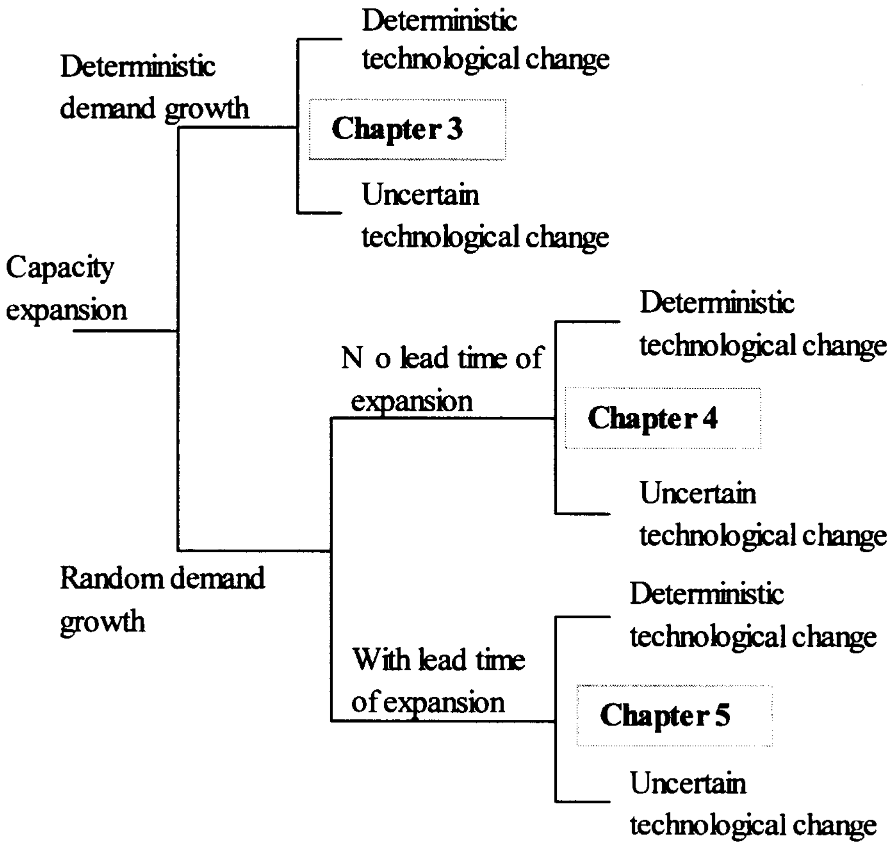

Figure 1.1 Thesis organization chart

CHAPTER 2. LITERATURE REVIEW AND PROBLEM DEFINITION

2.1 Overview

In this chapter, we review the previous studies in the scope of this thesis. The review can be grouped into capacity expansion problems and models, uncertainty in demand growth, technological progress and its effect on the capacity expansion problem, and finally lead time of construction, which may prompt expansion before the existing capacity is fully utilized.

2.2 Capacity expansion

Since the late 1950s, many studies of capacity expansion problems have been conducted. Sinden (1960) studied the capacity expansion problem of certain facilities providing service for a growing population, e.g., a power plant, a transportation system, a telephone system, etc. Sinden assumed the demand for services as a function of time is given; the facility must expand and replace its equipment from time to time in order to meet its demand. Finally, Sinden showed in certain cases, that there is an optimal expansion policy with equal time intervals between successive expansions. Manne (1961) studied the capacity expansion problems with probabilistic growth. His capacity expansion models also included the penalties involved in accumulating backlogs of unsatisfied demand. The result showed that uncertainty in demand growth causes larger size of capacity expansion, thus, higher expected discounted cost of expansion. Another study by Manne (1967) of several heavy process industries in India is an example widely known for its application. Manne's capacity expansion model consists of deterministic demand, which grows linearly over time. Suppose the capacity, once installed, has an infinite economic life and whenever demand reaches the existing capacity level, the capacity is expanded. Figure 1.1 illustrates Manne's capacity expansion model.

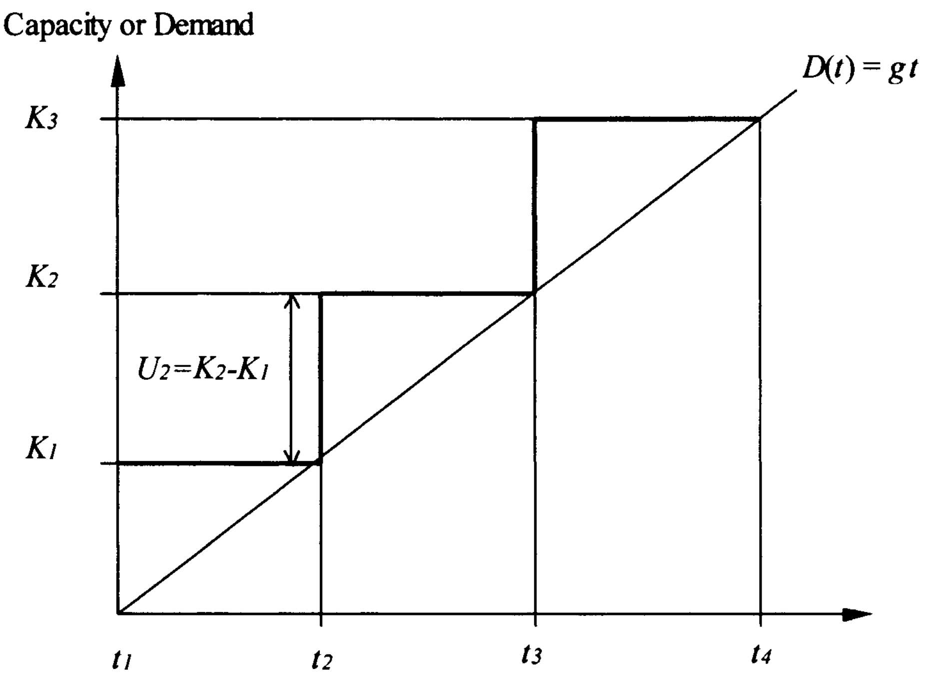

Figure 2.1 Capacity expansion process with linear demand growth.

The total cost of expansion over an infinite time horizon can be simply calculated by the summation of the cost of each replenishment discounted back to time $t = 0$. Srinivasan (1967) extended Manne's work to the growth of heavy industries in India. He formulated a model in which demand grows at a constant geometric rate, and assumed that there are no demand backlogs (excess demand). With the economies of scale in construction incorporated into the capacity expansion cost, it is optimal to expand capacity at each of a sequence of equally spaced of time points. Therefore, the optimal expansion size would grow exponentially. Srinivasan also assumed that technology is static over the problem horizon. The construction lead time for adding new capacity is assumed to be zero. A survey of Luss (1982) can be consulted as an extensive literature review on capacity expansion. In his survey, Luss unified the existing literature, emphasizing modeling approaches, algorithmic solutions and relevant application.

2.3 Technological progress

Several studies include the effect of technological progress on the capacity expansion models. Snow (1975) reviewed the previous work of Manne and Srinivasan (1967) and included a technological progress parameter in the capacity expansion model of the communications satellite INTELSAT. This added parameter is the annual exponential rate at which prices fall due solely to the effect of technological progress. Snow showed that the newly added technological change can affect the capacity expansion model by decreasing the discounted cost of each replenishment. More detail of Snow's work will be discussed in Chapter 3. Other previous studies that suggested the importance of technological change to the decision making in a capacity expansion or replacement problem are listed as follows. Goldstein et al. (1988) studied the effect of technological breakthrough on the machine replacement problem. They presented a dynamic discounted cost model and a method for finding the optimal age for replacement of an existing machine in a technological development environment. In their research, they assumed that a new technological breakthrough is about to enter the market in the form of new machine, which has higher purchased cost, but lower maintenance costs than the existing machine. Hopp and Nair

(1991) developed a procedure for computing the optimal replacement decision in an environment of technological change. Their model assumed that the costs associated with the present and future technologies are known, but the appearance times of the future technologies are uncertain. Nair (1995) studied the uncertain sequential technological change, which affects the firm's strategic investment decisions. He suggested that the appearance of the future technologies are considered uncertain with probabilities that may vary with time, but the order in which they appear is assumed sequential, like the different generations of microchips for personal computers. Finally, he developed an approach using nonunique terminal rewards to solve the dynamic programming model of the replacement problem. All the previous works discussed above show how important of technological change to the capacity expansion problem. The prediction and forecasting of technological change itself was shown in the research of Porter et al. (1991), which discussed the models of technology growth from the previous works by Gompertz and Fischer-Pry. Porter et al. suggested that the growth in capacity of many technologies is exponential over a considerable time period. Rajagopalan et al. (1998) formulated a capacity expansion and replacement model with a sequence of technological breakthroughs. They modeled the stochastic technological change as a semi-Markov process by specifying the distribution of the time between two consecutive innovations and the matrix of transition probabilities for the levels of technology achieved. The timing between innovations can accommodate the realistic situations by proper choice of time-to-discovery distribution which is different in many industries.

2.4 Uncertain demand growth

Various models have been formulated for the capacity expansion problem with random demand. Freidenfelds (1980) studied the effects of uncertainty in demand on capacity expansion decisions. He specified demand as a birth and death process for fixed expansion increment. Freidenfelds showed that the effect of randomness is identical to the effect of a larger growth rate. Davis et al. (1987) formulated the demand model as a random point process, i.e., increasing by discrete amounts at random times. Their capacity expansion model also included a cost associated with failure to meet demand, and a wasteful cost of overcapacity. They studied the problem by methods of stochastic control and presented a numerical algorithm to determine the optimal policy.

Bean, Higle and Smith (1992) showed that the capacity expansion problem over an infinite horizon with demand that follows either a nonlinear Brownian motion or a non-Markovian birth and death process could be transformed into an equivalent deterministic problem. This equivalent deterministic problem is formed by replacing the stochastic demand by its deterministic trend and replacing its cost discount factor by a new deterministic equivalent interest rate, which is smaller than the original, in approximate proportion to the uncertainty in the demand. More details of this study will be discussed in Chapter 4.

2.5 Lead time of expansion

There are some studies of capacity expansion model with lead time for adding capacity. Nickell (1977) formulated a model with an uncertain future change in demand and showed that the existence of a fixed lead time for adding new capacity would cause a firm to introduce a capacity increase earlier. He also showed that a longer lead time results in earlier anticipation of demand increases. Davis et al. (1987) presented a more mathematical model of the capacity expansion process of large scale projects that incorporated a controllable non-zero lead time of constructing new capacity into uncertain future demand forecast model. They formulated the optimization problem by methods of stochastic control theory, and finally presented numerical algorithms for finding an optimal policy and showed the solutions for some simple cases.

Chaouch and Buzacott (1994) studied the effect of lead time on the timing of plant construction with the objective of minimizing the expected discounted costs of expansions and shortages over the infinite time horizon. Their model has certain fixed lead time of construction that is independent of plant size. The demand grows alternately with constant rate in some periods and stagnant growth in other periods as illustrated in Figure 2.3. They suggested that it may be economically attractive to defer plant construction beyond the time when existing capacity became fully absorbed for a certain lead time duration. Longer lead times increase the optimal capacity trigger levels and sizes of capacity additions. Ryan (2000) formulated a dynamic programming model of capacity expansion for uncertain exponential demand growth and deterministic expansion lead times, and used option pricing formulas to estimate the shortages to result from a capacity expansion policy. Ryan showed that, with the expected lead time shortage fixed, the discounted expansion cost could be minimized by expanding capacity by a constant multiple of existing capacity.

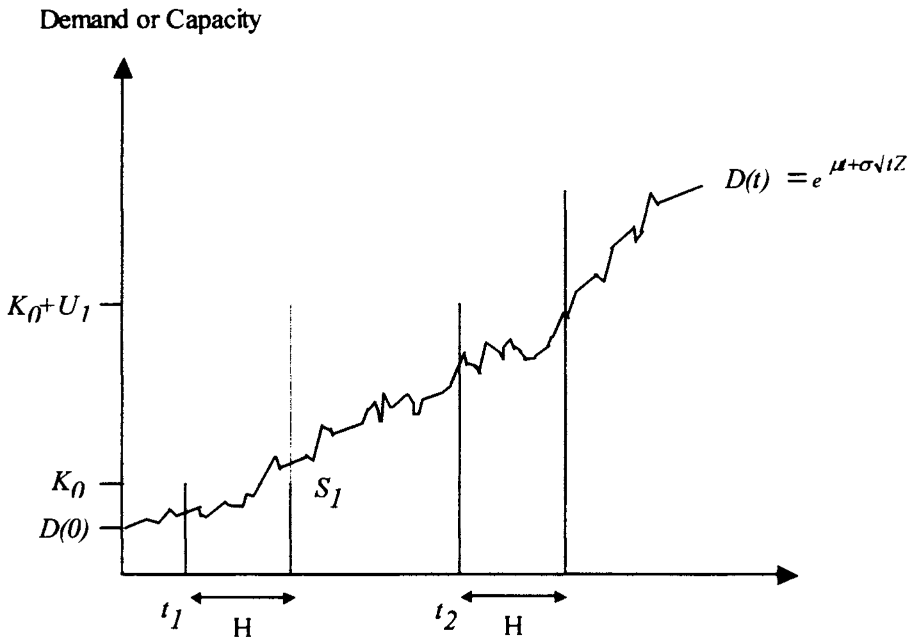

Figure 2.2 Expansion policy with fixed lead time H

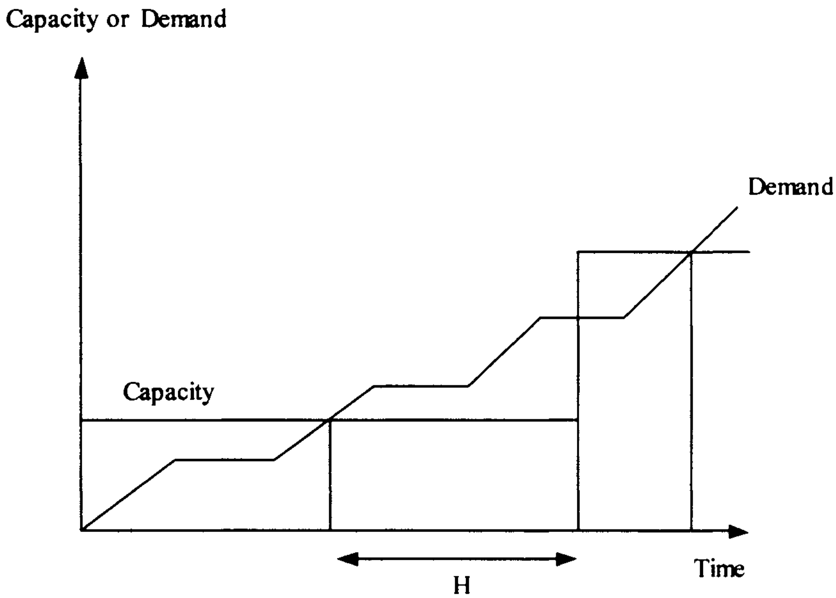

Figure 2.3 Chaouch and Buzacott's expansion policy

Pak (2001) suggested that the shortage estimation methods during the lead time of capacity expansion could be estimated as the values of some financial options. He modeled the capacity expansion with lead time of adding new capacity that creates the potential capacity shortage. Pak investigated four options including European call option, Asian option, Lookback option, and Summing European option. After comparison between these four financial options, he suggested that the Summing European Option with a proper subdivision of the lead time is the most appropriate estimate for lead time shortage. Figure 2.4 illustrates the concept of using the Summing European Option to estimate the potential shortage. More details will be discussed in Chapter 5.

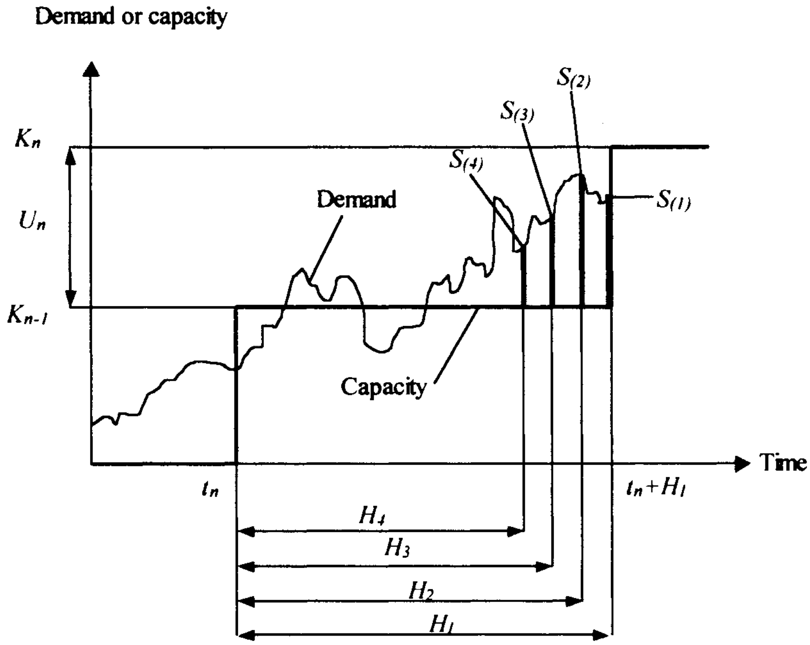

Figure 2.4 Shortage estimation using Summing European Option.

2.6 Notations

In this section, we list the common parameters being used in this thesis and their description. The parameters are grouped logically, according to whether they relate demand model, capacity cost, technological progress, or represent decision variables.

Demand model notations

g : Exponential rate of demand growth

$D(t)$: Demand of capacity at time t

μ : Constant rate (drift) of logarithmic demand growth

σ : Constant volatility of logarithmic demand growth

t_n: Time at n^{th} expansion

Capacity and cost notations

r : Annual interest rate

a : Economies of scale factor

n : Index of sequence of expansions $(n = 1,2,...)$

K_n: Installed capacity after n additions are completed

K_0: Initial capacity

$U_n = K_n - K_{n-1}$: Size of n^{th} capacity expansion

C : Total discounted cost of expansion

$E(C)$: Total expected discounted cost of expansion

H : Fixed installation lead time

$S_n^{(E)}$: Capacity shortage estimated by European option

S_n : The total shortage expected based on specific time unit during the lead time

$\Phi(\cdot)$: The standard normal cumulative distribution function

$u(\gamma, X)$: Infinite horizon expected capacity expansion cost

$v(\gamma, X)$: Infinite horizon expected discounted shortage

m: Penalty for shortage

Technological progress notations

p : Average cost decrease due to deterministic technological change

$\hat{p} = (1 - e^{-q})\lambda$: Equivalent cost decrease rate per year

λ : Average innovation per year

q : Average price drop per innovation

$N(t)$: A number of innovations up to an arbitrary time t.

Decision variables notations

γ : Expansion timing variable (ratio of potential shortage to existing capacity)

X : Expansion size variable

CHAPTER 3. DETERMINISTIC DEMAND GROWTH WITH TECHNOLOGICAL PROGRESS

3.1 Introduction

In this chapter, we focus on a model with deterministic demand growth and deterministic or uncertain technological change. We review and extend Snow's (1975) study of the impact of technological progress on satellite communication, which indicated that the appearance of technological improvements would cause a significant drop in the per unit cost of satellite communication. This drop is equivalent to an enlargement of the discount factor used in the capacity expansion cost function.

3.2 Deterministic technological change

The capacity expansion problem with deterministic demand growth has been widely discussed in many previous studies. Snow (1975) modified Srinivasan's (1967) capacity expansion model by assuming an exponential decrease in future capacity costs. Snow's example of the satellite communication system cost showed that new technology introduced to the system can cause a decrease in per unit price of the system. The exponential decrease in future capacity costs can be shown as

$$C(x,t+\Delta t)=e^{-p\Delta t}C(x,t)$$

where $C(x,t)$ is the discounted expansion cost of capacity size x at time t, and p denotes the cost decrease rate due to technological change.

Let $D(t)$ be the demand with annual exponential growth rate g. Let K_0 be the capacity level at time $t = 0$. Therefore, demand will reach a level of $K_0 e^{gt}$ at time t. The unit cost of adding U units of capacity at time 0 is assumed to equal kU^a current dollars, where a is an economies of scale parameter such that $0 < a < 1$. Throughout the thesis, we assume that cost units have been scaled so that the constant k is unity for simplicity. The cost of the same size expansion at time t is $e^{-\rho t}U^a$.

Since demand must always be met, the additional capacity will be installed once demand catches up to the existing capacity level. The problem is to determine the sequence of expansion sizes $\{U_n, n \geq 1\}$ that minimize the infinite horizon discounted cost.

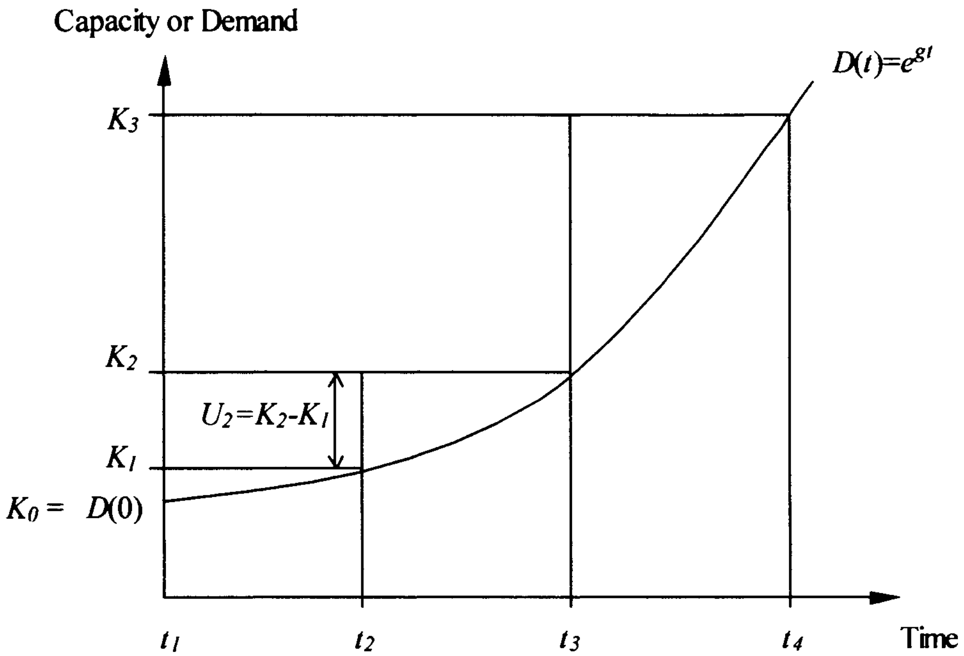

Figure 3.1 Exponential demand growth and expansion policy

From figure 3.1, the first replenishment U_1 can be written as the difference of installed capacity levels,

$$U_1 = K_1 - K_0$$
$$= K_0 e^{g(t_1 - t_0)} - K_0$$
$$= K_0 \left(e^{g(t_1 - t_0)} - 1\right)$$

Let e^{-rt} be the present value of one dollar at time t years, where r denotes the annual interest rate. Then, the total discounted cost of expansion can be summed.

$$C = \sum_{n=1}^{\infty} e^{-rt_n} e^{-pt_n} U_n^{\ a} = \sum_{n=1}^{\infty} e^{-(p+r)t_n} U_n^{\ a}$$

Therefore, the cost impact of technological change is equivalent to an increase in the interest rate. Sinden (1960) proved that the total cost of expansion will be minimized if the time between each pair of replenishment points is equal, i.e. $t_{n+1} - t_n = t_n - t_{n-1}$ for all $n \geq 2$. Let $\Delta t = t_n - t_{n-1}$, then

$$U_1 = K_1 - K_0$$
$$= K_0 e^{g(t_1 - t_0)} - K_0$$
$$= K_0 e^{g\Delta t} - K_0$$
$$= K_0 \left(e^{g\Delta t} - 1\right)$$

For convenience, let $G(\Delta t) = K_0 \left(e^{g\Delta t} - 1\right)$.

At time t_n, the replenishment size will be

$$U_n = K_0 e^{gt_{n+1}} - K_0 e^{gt_n},$$

or

$$U_n = e^{gt_n} G(\Delta t) \qquad (3.1)$$
$$= e^{gn\Delta t} G(\Delta t)$$

Hence, we have the cost function,

$$C = \sum_{n=1}^{\infty} e^{-rn\Delta t} e^{-pn\Delta t} \left(e^{gn\Delta t} G(\Delta t)\right)^a$$

$$= \sum_{n=1}^{\infty} e^{-(r+p-ag)n\Delta t} G(\Delta t)^a$$

By summing the infinite geometric series, the cost can be written in closed form;

$$C = \frac{G(\Delta t)^a}{1 - e^{-(r+p-ag)\Delta t}}, \qquad (3.2)$$

with the condition for convergence, $r + p - ag > 0$.

From this total cost function, we can find the optimal expansion policy parameter Δt by minimizing the function respect to Δt. We used the *FindMinimum* function in *Mathematica*, which searches for a local minimum (Wolfram, 1999, pp 1135).

As a baseline we used parameter values $g = 0.05$ (annual growth rate), $r = 0.1$ (annual risk-free interest rate), and $a = 0.7$ (economies of scale factor), and set the initial demand and capacity $K_0 = 1$ for convenience.

Figure 3.2 illustrates the effect of technological progress on the optimal expansion policy parameter or time interval between each expansion, when levels of the cost decrease rate varied from 0.01 to 0.2. As the cost decrease rate, p, becomes higher (i.e. introduction of a new model causes a large price drop of the latest product model), the optimal policy is to expand more frequently. Equivalently, expansion sizes become smaller as the cost impact of technological change increases.

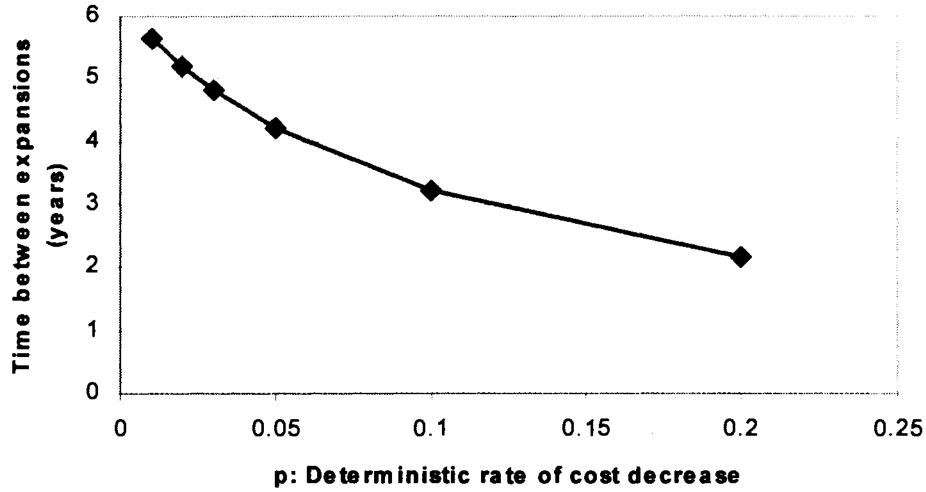

Figure 3.2 Effect of p: deterministic rate of cost decrease due to technological change

3.3 Uncertain Technological Change

In the previous section, we reviewed Snow's formulation of the capacity expansion problem with an assumption that technological progress is deterministic. The deterministic progress means we know the exact time when new technology will be introduced to the market. The newer technology creates better product efficiency, and has a direct impact on the present worth cost of expansion. For some types of capacity such as a computer's CPU, we definitely know that a newer CPU type means faster speed of calculation with better efficiency. However, we might not know the exact date of the introduction of that new CPU to the market. A more realistic model should consider the appearance times of newer technology to the market as an uncertain process.

In this section, we consider the capacity expansion problem with uncertain technological change. We assume the technological innovations follow a Poisson process. One possible example of Poisson technological change can be found in the medical imaging equipment industry over the past two decades. Successive of technologies such as X-ray, computed tomography (CT), magnetic resonance imaging (MRI), and positron emission tomography (PET) have provided better diagnostic information that has helped physicians in eliminating expensive surgeries and in selecting more appropriate medical therapies (Rajagopalan, 1998). Let λ denotes the average rate of innovations per unit time. Let $N(t)$ be the number of innovations up to an arbitrary time t. Let q be the exponential rate of cost decrease for each innovation. Further assumptions for the Poisson process are as follow:

1. Technology changes occur one at a time.
2. $N(t+\Delta t)-N(t)$, The number of technological changes during the interval $(t,t+\Delta t)$ is independent of $N(u)$, $0 \le u \le t$.
3. The distribution of $N(t+\Delta t)-N(t)$ is independent of t for all $\Delta t \ge 0$.

As in the deterministic technological change, the decrease in future capacity cost can be written as

$$C(x,t+\Delta t)=e^{-qN(\Delta t)}C(x,t),$$

Recall the capacity expansion cost function with deterministic technological change;

$$C=\sum_{n=1}^{\infty} e^{-rt_n} e^{-pt_n} U_n^{\ a}$$

In the uncertain technological change case, we assume that the cost of an expansion of size U at time t is $e^{-qN(t)}U^a$. Here, q denotes the rate of cost decrease per each innovation, and

λ denotes the average number of new innovations per year. We have the discounted cost function as

$$C = \sum_{n=1}^{\infty} e^{-rt_n} e^{-qN(t_n)} U_n^{\ a}$$

The expected discounted cost function is

$$E[C] = \sum_{n=1}^{\infty} e^{-rt_n} E\left[e^{-qN(t_n)}\right] U_n^{\ a} \qquad (3.3)$$

For any t, since $N(t)$ is a Poisson random variable, we can use the Poisson moment generating function $E\left[e^{-qN(t)}\right] = e^{-\left(1-e^{-q}\right)\lambda t}$ (Ross, 1985, p. 60), and obtain the equivalent deterministic cost decrease rate of $\hat{p} = \left(1 - e^{-q}\right)\lambda$. Thus, the total expected discounted cost function (3.3) is transformed to

$$E(C) = \sum_{n=1}^{\infty} e^{-rt_n} e^{-\left(1-e^{-q}\right)\lambda t_n} U_n^{\ a} .$$

From the definition of U_n in equation (3.1), $U_n = e^{gt_n} G(\Delta t)$, we then have

$$E(C) = \sum_{n=1}^{\infty} e^{-(r+\hat{p})n\Delta t} \left(e^{gn\Delta t} G(\Delta t)\right)^a$$

$$= \sum_{n=1}^{\infty} e^{-(r+\hat{p}-ag)n\Delta t} G(\Delta t)^a$$

Then, once again summing the geometric series

$$E(C) = \frac{G(\Delta t)^a}{1 - e^{-(r+\hat{p}-ag)\Delta t}} \qquad (3.4)$$

with the condition for convergence, $r + \hat{p} - ag > 0$.

As in the previous section, we can find the optimal expansion policy parameter Δt by minimizing the cost function (3.4) respect to Δt. We used the same baseline parameter values of g, r, and a, as before and, in addition, $\lambda = 0.5$ (average rate of innovation per year), and $q = 0.05$ (average cost decrease rate per innovation).

Figure 3.3 illustrates the effect of the innovation rate on the optimal policy parameter, Δt, when the level of the annual innovation rate varied from 0.2 to 2. At a high average rate (new technology occurs in the market more frequently), the optimal policy tends to encourage earlier and more frequent capacity expansion, i.e., the time interval between replenishments is shorter.

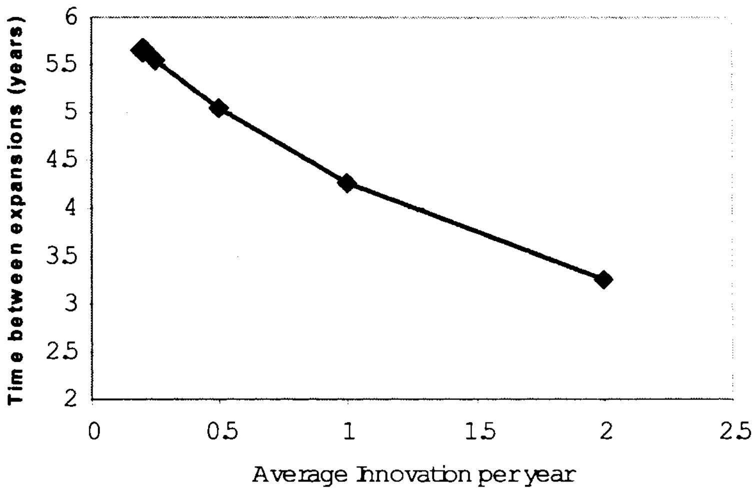

Figure 3.3 Effect of λ, innovation rate

Figure 3.4 illustrates the effect of q, the cost decrease rate per innovation, on the optimal policy parameter, when level of cost decrease rate varied from 0.01 to 0.1. As the cost decrease rate increase (product cost drops more dramatically if newer technology introduced), the optimal policy tends to encourage earlier capacity expansion and the time interval between expansions is shorter.

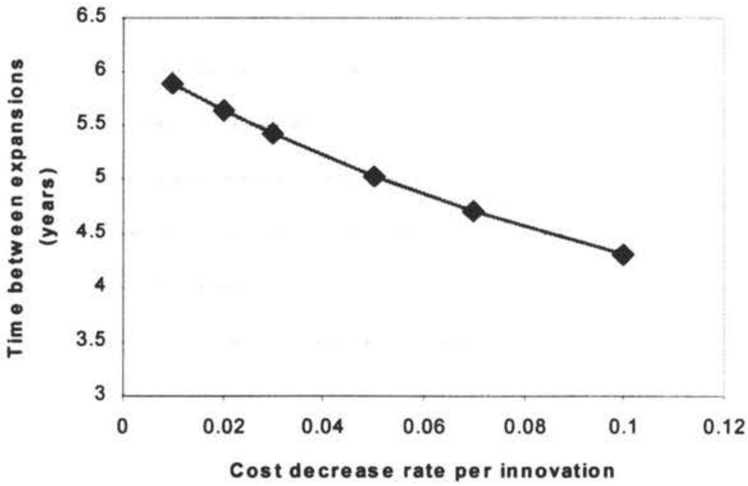

Figure 3.4 Effect of q, cost decrease rate per innovation

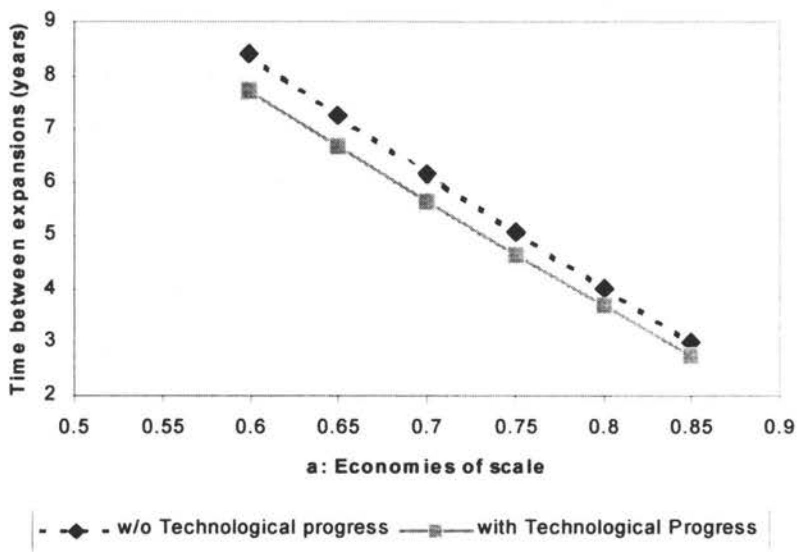

Figure 3.5 Comparing between the model with technological progress and the model without technological progress at various a, economies of scale factor

Figure 3.5 illustrates the effect of a, economies of scale parameter on the optimal expansion policy parameter of two capacity expansion models, one with technological progress, and another without technological progress, at the same baseline parameter values. Increasing values of a correspond to decreased economies of scale. As economies of scale increase (smaller a value), the optimal time interval between each expansion is increased for both capacity expansion models. Figure 3.5 shows that the appearance of technological progress apparently affects the optimal policy parameter by encouraging earlier expansion time. The capacity expansion problem in a technological progress environment is solved by incorporating an additional discount factor due to technological change. Earlier expansion or shorter time interval between each expansion means a smaller size of expansion, hence more flexibility to adopt products with newer technology.

Figures 3.6 and 3.7 illustrate the relation between parameter q (cost decrease rate per innovation), λ (annual rate of innovation), and $\hat{p}$ (equivalent cost decrease rate per year). The graphics show that q, the cost decrease rate per innovation, tends to have slightly lower impact to equivalent cost decrease rate per year than those of λ, the annual rate of innovation. In Figure 3.6, increasing the value of q by 100 percent from 0.01 to 0.02 causes the average value of $\hat{p}$ to increase by 49.2 percent, while increasing the value of λ by 100 percent from 1.0 to 2.0 causes an average $\hat{p}$ increase of 50.0 percent.

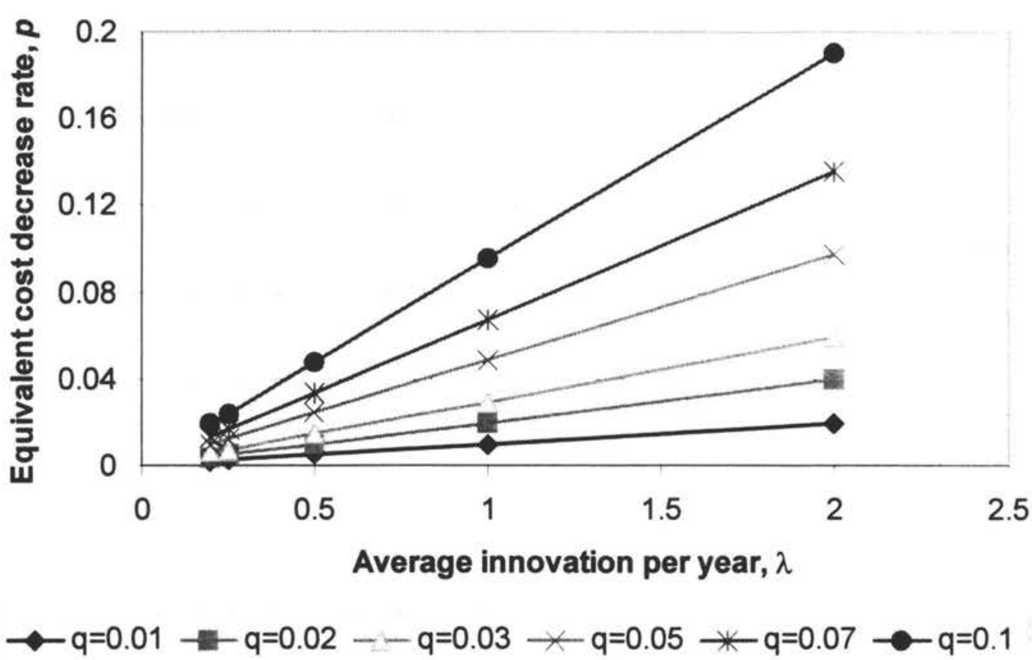

Figure 3.6 Relation of technology parameters q , λ , and $\hat{p}$.

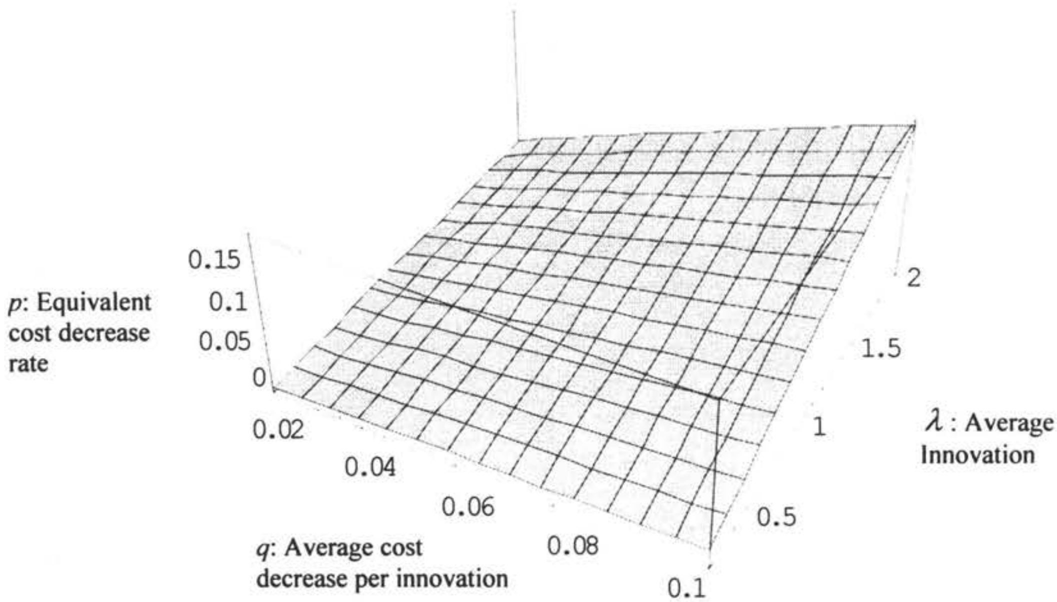

Figure 3.7 The 3-Dimension plot of the relation between parameters q , λ , and $\hat{p}$

CHAPTER 4: RANDOM DEMAND GROWTH WITH TECHNOLOGICAL PROGRESS

4.1 Introduction

In this chapter, we consider a capacity expansion model that has random demand growth and technological change. We consider both deterministic rate of technological change and uncertain occurrences of technological change. In this chapter, we assume there is no lead time for expansion, i.e., the manager can wait until demand reaches the existing capacity level and then add more capacity instantaneously. Also, there will be no risk of penalty due to capacity shortage in this scenario. Figure 4.1 illustrates the random demand growth pattern and its expansion policy. Here $X(t)$ is Brownian motion with drift $\mu > 0$, and variance $\sigma^2 > 0$, and $X(0) = 0$.

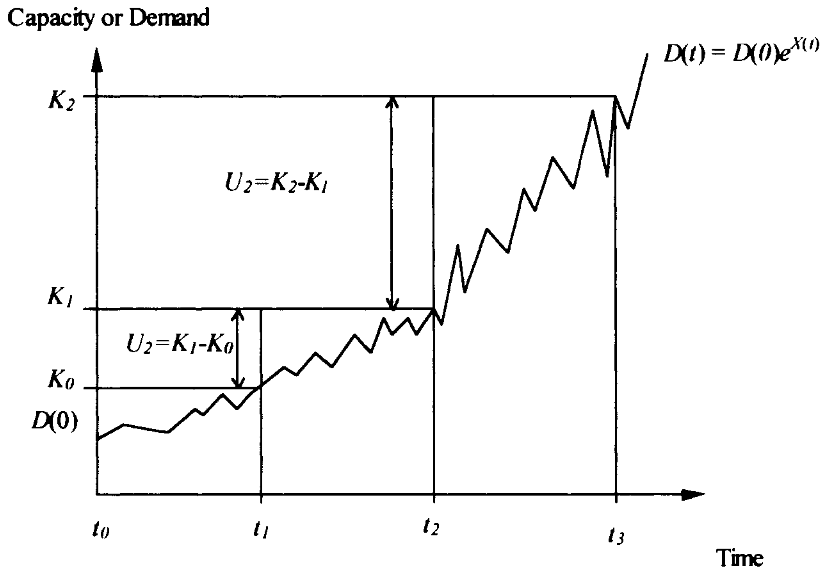

Figure 4.1 Random demand growth and expansion policy

We assume an exponential growth in demand for capacity with parameters μ as the constant drift of logarithmic demand growth, σ denotes constant volatility of logarithmic demand growth, and $g = \mu + \sigma^2 / 2$ as the mean exponential growth rate of demand. Let $D(t)$ denote the demand for capacity at time t, this demand satisfies

$$Log\left(\frac{D(t+\Delta t)}{D(t)}\right) = \mu\Delta t + \sigma\sqrt{\Delta t}Z,$$

where Z is a standard normal random variable that is independent of the demand for capacity, $D(t)$. Then it follows that, given the demand at time t, the demand at time $t+\Delta t$ is lognormally distributed with mean and variance given by:

$$E[D(t+\Delta t)|D(t)] = D(t)e^{g\Delta t}$$

$$Var[D(t+\Delta t)|D(t)] = D(t)^2 e^{2g\Delta t}\left(e^{\sigma^2\Delta t} - 1\right)$$

4.2 Deterministic Equivalent Problem

The previous study of Bean, Higle, and Smith (1992) suggests that, in certain cases, the problem of capacity expansion with stochastically growing demand can be transformed into an equivalent deterministic problem. Then, we can apply previous results for the deterministic problem to solve for the optimal expansion policy.

Let $T(x)$ be the time at which a total capacity x is exhausted. In specifying the deterministic formulation, the discount factor for costs incurred at the random time $T(x)$ is replaced by its expected value, that is $e^{-rT(x)}$ is replaced by $E\left[e^{-rT(x)}\right]$. A nonnegative number r^*, such that $E\left[e^{rT(x)}\right] = e^{-r^* E[T(x)]}$ for all $x \geq 0$, is said to be an equivalent interest rate for the deterministic problem. When an equivalent interest rate r^* exists, the original stochastic capacity expansion problem may be solved via a deterministic problem formulation in which

a. The random expansion times $T(x)$ are replaced by their expected values.

b. The original interest rate is replaced by its equivalent r^*.

By Jensen's inequality, it follows that $r^* \le r$ (Freidenfelds 1981, pp.41). The effect of demand uncertainty is summarized by a drop in the effective rate of interest. The optimal expansion size would be larger than in the absence of uncertainty. Without lead times for adding capacity, if demand is a transformation of Brownian motion, the following theorem says that the problem with uncertain demand can be solved deterministically.

Theorem (Bean et al. 1992)

If $\{P(t), t \ge 0\}$ is transformed Brownian motion with underlying drift $\mu > 0$ and variance σ^2, and transforming function h, then let $P^*(t) = h(\mu t)$ and

$$r^* = \left(\frac{\mu}{\sigma}\right)^2 \left(\sqrt{1 + 2r\left(\frac{\sigma}{\mu}\right)^2} - 1\right).$$

Every optimal capacity expansion sequence for the deterministic problem with demand $P^*(\cdot)$ in which all costs are continuously discounted using the interest rate $r*$ is optimal for the stochastic problem.

For linear Brownian motion with drift $\mu > 0$ and variance σ^2, let $T(x) = \inf\{t \ge 0 : X(t) \ge x\}$ be the time at which a total capacity x is exhausted. Then

$$E\left[e^{-rT(x)}\right] = e^{-\rho x}, \tag{4.1}$$

where $\rho = \frac{\mu}{\sigma^2}\left(\sqrt{1+2r\left(\frac{\sigma}{\mu}\right)^2} - 1\right)$(Karlin and Taylor 1975, pp.362).

Also,

$$E[T(x)] = -\frac{d}{dr}E\left[e^{-rT(x)}\right]_{r=0}$$

$$= -\frac{d}{dr}e^{-\frac{\mu x}{\sigma^2}\left(\sqrt{1+2r\left(\frac{\sigma}{\mu}\right)^2}-1\right)}\Bigg|_{r=0}$$

$$= -\left[e^{-\frac{\mu x}{\sigma^2}\left(\sqrt{1+2r\left(\frac{\sigma}{\mu}\right)^2}-1\right)}\frac{d}{dr}\left[-\frac{\mu x}{\sigma^2}\left(\sqrt{1+2r\left(\frac{\sigma}{\mu}\right)^2}-1\right)\right]\right]_{r=0}$$

$$= (-1)\left[-\frac{\mu x}{\sigma^2}\left(\frac{1}{2}\right)\left(1+2r\left(\frac{\sigma}{\mu}\right)^2\right)^{-1/2}2\left(\frac{\sigma}{\mu}\right)^2\right]_{r=0}$$

$$= \frac{\mu x}{\sigma^2}\left(\frac{1}{2}\right)(1)(2)\left(\frac{\sigma}{\mu}\right)^2$$

$$= \frac{x}{\mu}.$$

When an equivalent interest rate exists, it is unique and is given by (Bean et al., 1992, pp 212)

$$r^* = \frac{\left|\ln E\left[e^{-rT(x)}\right]\right|}{E[T(x)]} = \frac{\frac{\mu}{\sigma^2}\left(\sqrt{1+2r\left(\frac{\sigma}{\mu}\right)^2}-1\right)x}{x/\mu}$$

$$=\left(\frac{\mu}{\sigma}\right)^2\left(\sqrt{1+2r\left(\frac{\sigma}{\mu}\right)^2}-1\right)=\mu\rho$$

In this chapter, we assume the demand model follows a geometric Brownian motion with drift and can be written as

$$D(t)=D(0)e^{X(t)}.$$

Let $h(X(t))=D(t)=D(0)e^{X(t)}$ be the transforming function. Also, let $y=h(x)=D(0)e^{x}$.

Hence,

$$x=h^{-1}(y)=\ln\left(\frac{y}{D(0)}\right)$$

Let $\tau(y)=\inf\{t\geq 0: D(t)\geq y\}$ be the time when demand $D(t)$ reaches the capacity level y

$$\tau(y)=\inf\{t\geq 0: X(t)\geq h^{-1}(y)\}$$

$$=T(h^{-1}(y))$$

From (4.1) $$E\left[e^{-r\tau(y)}\right]=E\left[e^{-rT(h^{-1}(y))}\right]=e^{-\rho h^{-1}(y)},$$

and $$E[\tau(y)]=E[T(h^{-1}(y))]=\frac{h^{-1}(y)}{\mu}.$$

So, the equivalent interest rate is given by

$$r^*=\frac{\rho h^{-1}(y)}{h^{-1}(y)/\mu}=\left(\frac{\mu}{\sigma}\right)^2\left(\sqrt{1+2r\left(\frac{\sigma}{\mu}\right)^2}-1\right) \tag{4.2}$$

Let t_n be the time when demand reach existing capacity level K_{n-1}, i.e.,

$t_n=\inf\{t\geq 0: D(t)\geq K_{n-1}\}$.

Then

$$E\left[e^{-r t_n}\right] = E\left[e^{-r\tau(K_{n-1})}\right] = e^{-\rho h^{-1}(K_{n-1})}$$

$$= e^{-\rho \ln\left(\frac{K_{n-1}}{D(0)}\right)} = \left(e^{\ln\left(\frac{K_{n-1}}{D(0)}\right)}\right)^{-\rho}$$

$$= \left(\frac{K_{n-1}}{D(0)}\right)^{-\rho} = \left(\frac{D(0)}{K_{n-1}}\right)^{\rho}$$

The advantages of solving the stochastic problem through its deterministic equivalent are seen by applying the optimal policy used in Chapter 3. Suppose that capacity costs are fixed and only depend on the capacity size with economies of scale $0 < a < 1$. That is, suppose the cost $C(U)$ *of* providing capacity of size U is given by $C(U) = U^a$, where $0<U<\infty$. The optimal expansion policy is to install facilities of increasing capacities to result in equal time intervals T^* between each installation times. Let $D(0)$ be the initial demand for the capacity, and μ denote the annual demand growth rate. The total expansion cost function can be written as

$$C = \frac{\left(D(0)\left(e^{\mu t} - 1\right)\right)^a}{1 - e^{-\left(r^* - a\mu\right)T}}, \tag{4.3}$$

with the condition for convergence, $r^* - a\mu > 0$. And, T^* can be found by minimizing the cost function respect to T, or

$$T^* = \arg\min_{T>0} \frac{\left(D(0)\left(e^{\mu t} - 1\right)\right)^a}{1 - e^{-\left(r^* - a\mu\right)T}}$$

With this optimal time interval between each installation, the optimal sequence of capacity additions is given by

$$U_n^* = D(0)e^{\mu(n-1)T^*}\left(e^{\mu T^*} - 1\right)$$

For $n = 1,2,3,\ldots$. Note that the effect of demand uncertainty as measured by σ^2 is completely included within r^*.

4.3 Equivalent deterministic problem with technological progress

In the previous section, we have developed the equivalent deterministic capacity expansion problem with the objective function or capacity expansion cost in equation (4.3). In this section, we apply the technological progress, which affects the total discounted cost of expansion as described in Snow (1975) into the equivalent deterministic model.

The expected discounted cost is $\sum_{n=1}^{\infty} E\left[e^{-rt_n} e^{-pt_n}\right] y_n^{\,a}$.

The discount factor is given by

$$E\left[e^{-rt_n} e^{-pt_n}\right] = E\left[e^{-(r+p)t_n}\right]$$

$$= E\left[e^{-r't_n}\right],\; r' = r + p$$

Since

$$E\left[e^{-r't_n}\right] = E\left[e^{-r'\tau(K_{n-1})}\right]$$

$$= E\left[e^{-r'T\left(h^{-1}(K_{n-1})\right)}\right]$$

it follows that,

$$E\left[e^{-r't_n}\right] = e^{-\hat{\rho} h^{-1}(K_{n-1})},$$

where

$$\hat{\rho} = \frac{\mu}{\sigma^2}\left(\sqrt{1 + 2r'\left(\frac{\sigma}{\mu}\right)^2} - 1\right)$$

and

$$h^{-1}(K_{n-1}) = \ln\left(\frac{K_{n-1}}{D(0)}\right).$$

Hence, the equivalent interest rate $\hat{r}_1^* = \mu\hat{\rho} = \left(\frac{\mu}{\sigma}\right)^2\left(\sqrt{1+2r'\left(\frac{\sigma}{\mu}\right)^2}-1\right)$ can solve the equivalent deterministic problem to $\min\sum_{n=1}^{\infty} e^{-\hat{r}_1^* h^{-1}(K_{n-1})}U_n^{\,a}$. The optimal policy parameter, T^*, can be found by

$$T^* = \arg\min_{T>0} \frac{\left(D(0)\left(e^{\mu T}-1\right)\right)^a}{1-e^{-\left(\hat{r}_1^* - a\mu\right)T}}$$

At base line we used parameter value $\mu = 0.05$ (average growth rate), $\sigma^2 = 0.1$ (variance associated with demand growth), and $a = 0.7$ (economies of scale factor).

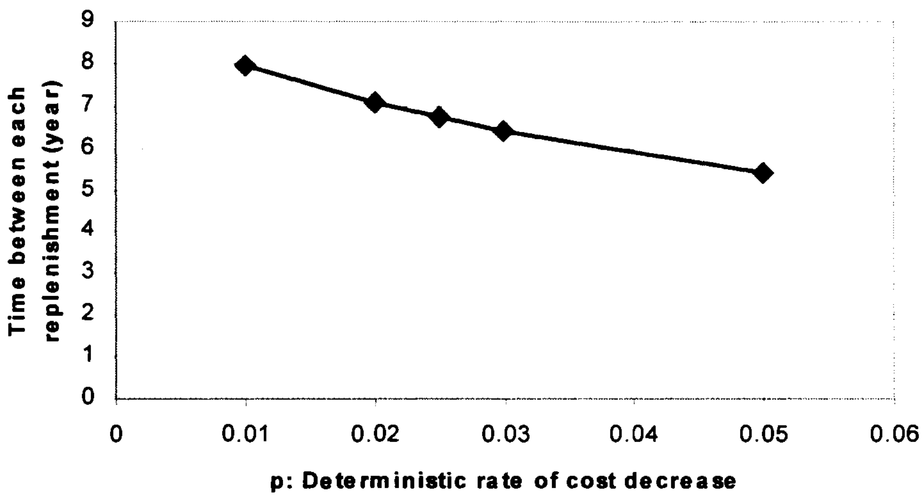

Figure 4.2 Effect of *p*, rate of cost decrease due to technological change

Figure 4.2 illustrates the effect of technological progress on the optimal expansion policy parameter or time interval between each expansion, when levels of cost decrease rate varied from 0.01 to 0.2. As the cost decrease rate, p, becomes higher (product price drops heavily when newer technology become available), the optimal policy encourages earlier expansion or a shorter time interval between each expansion. The result also indicates that timing policy is less sensitive to the rate of cost decrease at higher value of p.

4.4 Uncertain technological progress

In this section, we consider the capacity expansion problem with uncertain technological change i.e. we might know only the trend of technological changes. As in section 3.3, we assume the technological change as a Poisson process with rate λ, the average rate of innovation per unit time. Let $N(t)$ be a number of innovations up to an arbitrary time t. We assume that $N(t)$ is independent of the demand for capacity. Let q be the exponential rate of cost decrease for each innovation. The expected discounted cost is $\sum_{n=1}^{\infty} E\left[e^{-rt_n} e^{-qN(t_n)}\right] U_n^{\ a}$. The discount factor is given by

$$
\begin{aligned}
E\left[e^{-rt_n} e^{-qN(t_n)}\right] &= E_{t_n}\left[E\left[e^{-rt_n} e^{-q(t_n)} \middle| t_n\right]\right] \\
&= E_{t_n}\left[e^{-rt_n} E\left[e^{-qN(t_n)} \middle| t_n\right]\right] \\
&= E_{t_n}\left[e^{-rt_n} e^{\lambda t_n\left(e^{-q}-1\right)}\right] \\
&= E_{t_n}\left[e^{-\left(r+\lambda\left(1-e^{-q}\right)\right)t_n}\right] \\
&= E\left[e^{-\hat{r}(q,\lambda)t_n}\right], \quad \hat{r}(q,\lambda) = r + \lambda\left(1-e^{-q}\right) = r + \hat{p}
\end{aligned}
$$

Thus, we have

$$E\left[e^{-rt_n}e^{-qN(t_n)}\right] = E\left[e^{-\left(r+\lambda\left(1-e^{-q}\right)\right)t_n}\right]$$

$$= E\left[e^{-\hat{r}(\lambda,q)t_n}\right]$$

In this case, the equivalent interest rate

$$\hat{r}_2^* = \mu\hat{\beta} = \left(\frac{\mu}{\sigma}\right)^2\left(\sqrt{1+2\hat{r}\left(\frac{\sigma}{\mu}\right)^2}-1\right)$$

can solve the equivalent deterministic problem to $\min\sum_{n=1}^{\infty} e^{-\hat{r}_2^* h^{-1}(K_{n-1})} U_n^{\,a}$. The optimal policy parameter, T^*, can be found by

$$T^* = \arg\min_{T>0} \frac{\left(D(0)\left(e^{\mu T}-1\right)\right)^a}{1-e^{-\left(\hat{r}_2^*-a\mu\right)T}}$$

as in deterministic technological progress. We also use the same base line parameter value as those in section 4.3 as $\mu = 0.05$ (average growth rate), $\sigma^2 = 0.1$ (variance associated with demand growth), $a = 0.7$ (economies of scale factor).

Figure 4.3 illustrates the effect of the innovation rate on the optimal policy parameter, when the annual innovation rate varied from 0.2 to 2. At high average rate (new technology occurs in the market more frequently), the optimal policy tends to encourage earlier capacity expansion or the time interval between replenishment is shorter. The graphic result also shows that large economies of scale cause the shorter interval of time between each expansion.

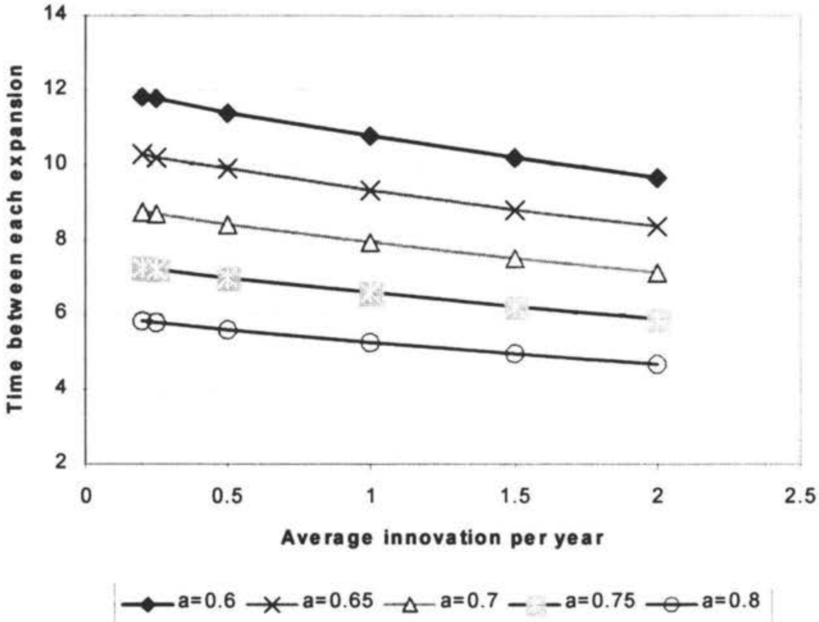

Figure 4.3 Effect of λ average innovation per unit time

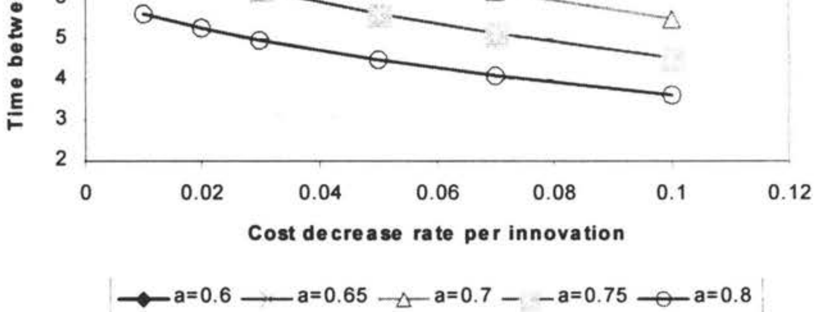

Figure 4.4 Effect of q, cost decrease rate per innovation

Figure 4.4 illustrates the effect of *q*, the cost decrease rate per innovation, on the optimal policy parameter, when level of cost decrease rate varied from 0.01 to 0.1. As the cost decrease rate increases (product cost drops significantly if newer technology introduced), the optimal policy tends to encourage earlier capacity expansion or the time interval between replenishment is shorter. The graphical result also shows that the optimal policy tends to be less sensitive to q at high values of q than at lower values of q. A large value of a, economies of scale, causes a shorter time interval between each expansion. When the value of q becomes higher, the optimal policy becomes less sensitive to the *a* value. The span of differences in optimal policy by varying a from 0.6 to 0.8 is smaller when the level of q is higher.

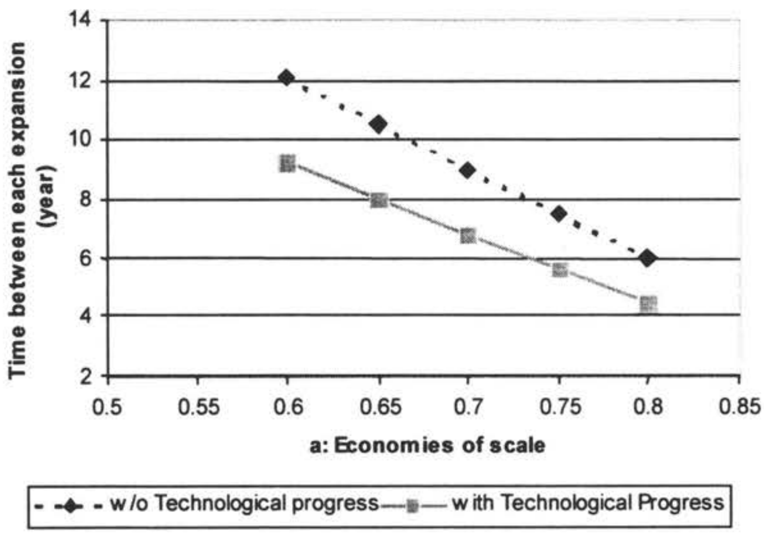

Figure 4.5 Comparing the problem with technological progress and problem without technological progress

Figure 4.5 shows that the appearance of technological progress apparently causes effect to the optimal policy parameter by encouraging a shorter time interval between each expansion time. As in the problem with deterministic demand growth in Chapter 3, capacity expansion problem in technological progress environment incorporates an additional discount factor due to technological change. Earlier expansion or shorter time interval between each expansion means smaller size of expansion, hence more flexibility to adopt products with newer technology.

CHAPTER 5: RANDOM DEMAND GROWTH WITH LEAD TIME FOR CONSTRUCTION

5.1 Introduction

In the previous chapters, we discussed the capacity expansion problem with technological change in several cases including deterministic and random demand growth with no lead time of installation. However, the existence of an installation lead time for adding new capacity, which occurs in many industries, can cause the risk of capacity shortage during that period. Therefore, the consideration of lead time in capacity expansion problems is appropriate in many actual situations. In this chapter, we focus on the capacity expansion model with random demand growth in the presence of a fixed installation lead time. With the potential shortage penalty caused by this lead time, the total expansion cost function discussed in Chapter 4 will be augmented with a shortage penalty cost. We formulate the problem in two cases; the first case features deterministic technological change, and the second case has uncertain technological change. Figure 5.1 illustrates the potential capacity shortage during the lead time of expansion, which occurs when demand growth during the lead time surpasses the existing level of excess capacity. Here, S_1 is the shortage during the lead time of expansion at time $t_1 + H$.

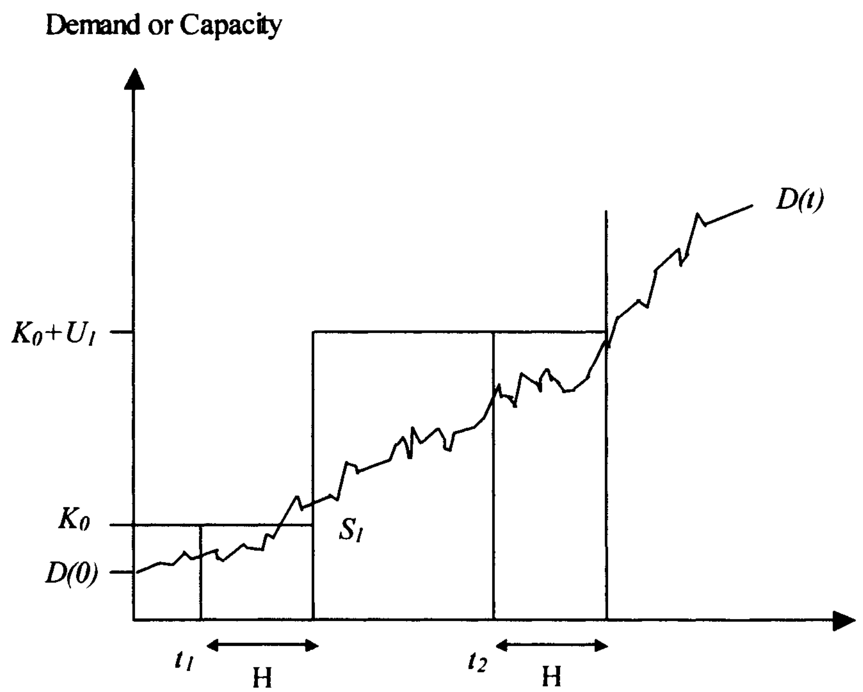

Figure 5.1 Capacity shortage during lead time of construction

5.2 Demand model and expansion policy

In addition to the demand model in Chapter 4, we assume that a fixed lead time of H time units is required to install additional capacity in any quantity. The cost of a capacity increment is assumed to be discounted continuously by interest rate $r > g$ and the rate of cost decrease due to deterministic or random technological change is as appears in the previous chapters.

Since we assume a lead time to install more capacity, there is the risk of capacity shortage during that installation time. This potential shortage differentiates the total cost function for the expansion models in this chapter from those in Chapter 3 and Chapter 4. In this chapter, the total cost of expansion consists of capacity cost and shortage penalty. There are two problems to deal with, in order to minimize the infinite horizon discounted expansion cost and control the risk of shortage during the lead time. The first problem is the timing of adding new capacity; the second problem concerns the size of the additional capacity. The optimal expansion policy discussed in this chapter addresses these issues.

In order to estimate the shortage cost, we need to estimate the shortage during the lead time of expansion. This shortage can be estimated by similarity with the value of financial options. According to Jarrow and Rudd (1983), a European call option is the right (but not the obligation) to buy an asset at a certain *strike* price on a certain *expiration* date. Birge (2000) showed that the risk in manufacturing and service operations decisions with limited capacity can be incorporated into the planning models by using the option pricing theory. He noted that "the plant has an option on the demand market as long as its capacity is not exceeded. Another way to view this situation is that the owner of the finite-capacity plant holds all of the demand but then sells off an option to other producers to capture any demand beyond the plant's capacity" (Birge, 2000 pp.20). Ryan (2000) noticed that the expected quantity of shortage at the end of lead time is mathematically identical to the value of a European call option on an asset. Demand for the capacity and the lead time of expansion can be compared to stock price and time to expiration, respectively. The expected quantity of shortage at the end of lead time using the Black-Scholes option pricing formula is given by (Jarrow and Rudd, 1983)

$$S_n^{(E)} = E\left[(D(t_n + H) - K_{n-1})^+\right] = e^{gH} D(t_n)\Phi(h) - K_{n-1}\Phi\left(h - \sigma\sqrt{H}\right)$$

where $h = \dfrac{\log(D(t_n)/K_{n-1}) + (g + \sigma^2/2)H}{\sigma\sqrt{H}}$

and $\Phi(.)$ is the standard normal cumulative distribution function.

Since the shortage is measured in terms of proportion to installed capacity, then

$$\frac{S_n}{K_{n-1}} = e^{gH}\left(\frac{D(t_n)}{K_{n-1}}\Phi(h) - \Phi\left(h - \sigma\sqrt{H}\right)\right)$$

depends on demand and installed capacity only their ration. Therefore, potential shortage can be controlled by a fixed ratio $\gamma = D(t_n)/K_{n-1}$, independent of *n*, which will trigger an expansion.

Timing policy: The n^{th} expansion installing time (t_n) is the minimum value of time, *t* when $D(t) = \gamma K_{n-1}$. Here, γ is a decision variable.

In electric utilities, before deregulation, this was known as a proportional reserve policy. A larger γ value means delaying the expansion until demand is close to the existing capacity level. This delay would cause a risk of capacity shortage during the lead time, although it might be economically advantageous. On the other hand, a smaller γ provides a smaller risk of shortage, but it would result in increased present value of capital cost.

As in previous chapters, the infinite horizon discounted expansion cost is the summation of the cost of a capacity increment of size U_n multiplied by the discount factor for the n^{th} expansion.

$$C = \sum_{n=1}^{\infty} E\left[e^{-rt_n}\right](U_n)^a \tag{5.1}$$

Theorem (Ryan (2000))

Supposed that the n^{th} expansion is initiated at the minimum value of t for which $D(t) = \gamma K_{n-1}$, where γ is fixed. Then the infinite horizon discounted expansion cost, C, is minimized by installing $U_n = K_n - K_{n-1}$ such that $K_n = (X+1)^n K_0$, where $X > 0$.

Assuming the timing policy is followed, the sizing policy can be derived from the above theorem.

Sizing policy: Under the timing policy, the n^{th} expansion is given by $U_n = XK_{n-1}$, for some $X > 0$.

The second decision variable, X, can be interpreted as the multiple of current capacity to install in each expansion, which means that the n^{th} expansion size is a fixed multiple of existing capacity from the previous expansions. To achieve an appropriate tradeoff between shortage risk and capital cost is the ultimate goal of optimizing the expansion policy parameters, γ and X.

5.3 Shortage estimation using summing European option

Pak (2001) noticed that the usage of European call option to estimate the capacity shortage is not completely satisfactory since it measures the shortage only at one instant at the end of the lead time. He compared several alternatives such as Asian option, Lookback option, and Summing European option and finally suggested the Summing European option as the most accurate and comprehensive shortage measure. The main idea of Summing European Option is to replace the fixed lead-time by a series of decreasing lead times and sum up all the expected shortages at the end of each smaller lead time as measured by the value of a European Option. The expected shortage from this smaller lead time should be discounted continuously back to time t_n because we measure expected shortage throughout the entire lead time. Figure 5.2 illustrates the idea clearly.

For a given series of lead times H_1, H_2, H_3, and H_4, The expected shortages, $S_{(1)}$, $S_{(2)}$, $S_{(3)}$, and $S_{(4)}$ can be calculated by the Black-Scholes formula. Let $\Delta t = H_i - H_{i+1}$. Then the total expected shortage during the n^{th} lead time, discounted continuously back to time t_n, can be approximated as follows (Pak, 2001 pp 26-27).

$$S_n = \int_{t_n}^{t_n+H} e^{-r(t-t_n)} E\left[(D(t) - K_{n-1})^+\right] dt$$

$$= \sum_{i=1}^{m} \int_{t_n+(i-1)\Delta t}^{t_n+i\Delta t} e^{-r(t-t_n)} E\left[(D(t) - K_{n-1})^+\right] dt,$$

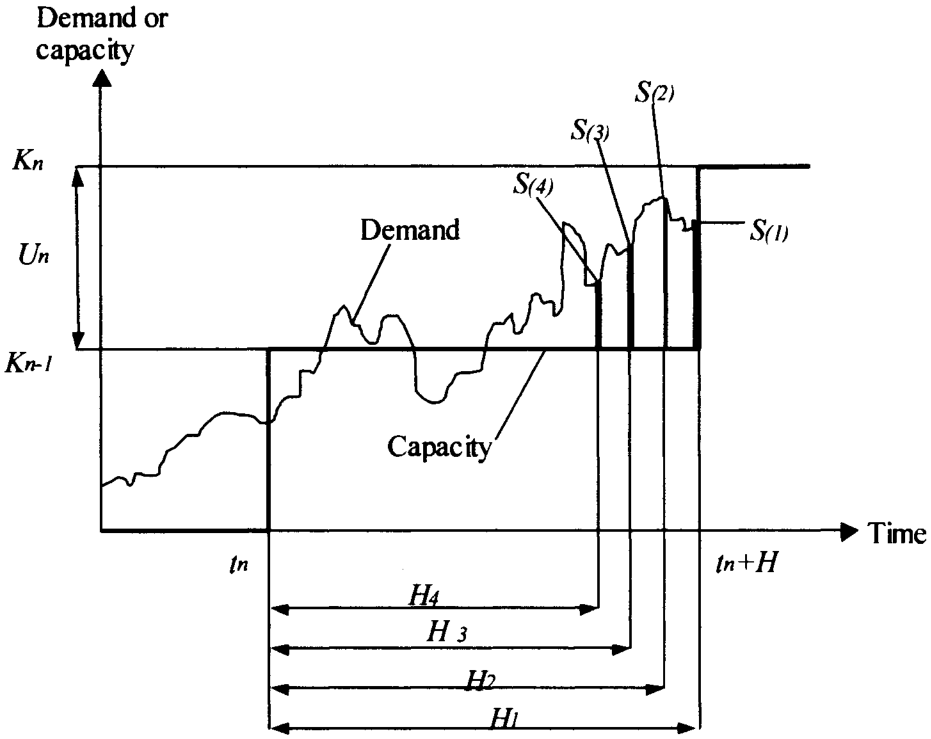

Fig 5.2 Series of lead times and shortages of Summing European Option

The lead time H, measured in years, is subdivided into m intervals. By a Riemann-integral approximation, we can bound the expected shortage as a proportion of existing capacity from above and below as

$$\frac{S_n}{K_{n-1}} \leq \sum_{i=1}^{H}\left(e^{-(r-g)i\Delta t}\frac{D(t_n)}{K_{n-1}}\Phi(h_i) - e^{-ri\Delta t}\Phi\left(h_i - \sigma\sqrt{H}\right)\right)\Delta t = UB,$$

$$\frac{S_n}{K_{n-1}} \geq \sum_{i=1}^{H}\left(e^{-(r-g)(i-1)\Delta t}\frac{D(t_n)}{K_{n-1}}\Phi(h_i) - e^{-r(i-1)\Delta t}\Phi\left(h_i - \sigma\sqrt{H}\right)\right)\Delta t = LB,$$

where $\Delta t = \dfrac{H}{m}$.

Recall that, when following the timing policy, $\dfrac{D(t_n)}{K_{n-1}} = \gamma$ (a constant).

The shortage estimate is found by averaging the lower and upper bounds,

$$\frac{S_n}{K_{n-1}} = \frac{S_1}{K_0} = \frac{1}{2}(LB + UB).$$

To obtain an accurate estimation of shortage with a reasonable amount of computation, Pak (2001) suggested the subdivision of lead time on daily basis: $\Delta t = \dfrac{1}{364}$ year, $m = \dfrac{H}{\Delta t} = 182$, when $H = 0.5$, as the compromising point between accuracy of the result and the computation time.

Let S_n be the expected shortage estimated by the Summing European Option method. We have the expected discounted shortage function over the infinite horizon as

$$v(y, X) = \sum_{n=1}^{\infty} E\left[e^{-rt_n}\right]S_n \tag{5.2}$$

As in Chapter 4, the expected value of the discount factor is given by

$$E\left[e^{-r t_n}\right] = E\left[e^{-r\tau(\gamma K_{n-1})}\right] = e^{-\rho h^{-1}(\gamma K_{n-1})}$$

$$= e^{-\rho \ln\left(\frac{\gamma K_{n-1}}{D(0)}\right)} = \left(e^{\ln\left(\frac{\gamma K_{n-1}}{D(0)}\right)}\right)^{-\rho}$$

$$= \left(\frac{\gamma K_{n-1}}{D(0)}\right)^{-\rho} = \left(\frac{D(0)}{\gamma K_{n-1}}\right)^{\rho} \tag{5.3}$$

where

$$\rho = \frac{\mu}{\sigma^2}\left(\sqrt{1 + 2r\left(\frac{\sigma}{\mu}\right)^2} - 1\right)$$

Notice that the discount factor $E\left[e^{-rt_n}\right] = \left(\frac{D(0)}{K_{n-1}}\right)^{\rho}$ in Chapter 4 is similar to the discount factor in (5.3) except that $\gamma = 1$ because in the capacity expansion model in Chapter 4, the manager can wait until demand reaches the existing capacity level before installing additional capacity. From (5.2), we have policy parameters γ and X, the multiple of current capacity to install at each expansion. This shortage function can be written as

$$v(\gamma, X) = \sum_{n=1}^{\infty}\left(\frac{D(0)}{\gamma K_{n-1}}\right)^{\rho} S_n$$

$$= \left(\frac{D(0)}{\gamma}\right)^{\rho} \sum_{n=1}^{\infty} \frac{S_n}{K_{n-1}} K_{n-1}{}^{1-\rho}$$

$$= \left(\frac{D(0)}{\gamma}\right)^{\rho} \left(\frac{S_1}{K_0}\right) \frac{K_0{}^{1-\rho}}{1-(X+1)^{1-\rho}}$$

and can be restated as

$$v(\gamma, X) = \left(\frac{D(0)}{K_0}\right)^{\rho} K_0 y(\gamma, X), \tag{5.4}$$

where

$$y(\gamma, X) = \left(\frac{S_1}{K_0}\right) \frac{\gamma^{-\rho}}{1-(X+1)^{1-\rho}} \tag{5.5}$$

5.4 Discounted cost factor due to technological progress

Recall the capacity cost function (5.1),

$$C = \sum_{n=1}^{\infty} E\left[e^{-rt_n}\right](U_n)^a \quad .$$

The discount factor for the n^{th} expansion is given by

$$E\left[e^{-rt_n}\right] = \left(\frac{D(0)}{\gamma K_{n-1}}\right)^{\rho}$$

As we discussed in Chapter 2, technological change can affect the total cost of expansion by decreasing the present value of the discounted cost of expansion over the infinite time horizon. In this section, we consider the effect of deterministic of random technological change into two categories by its nature of appearance. The first category is technological change with deterministic rate. The example of this category is the capacity expansion model of the communications satellite system in Snow (1975)'s study. The other is uncertain technological change, in which the exact rate of technological change is not known. Example of this category can be found in Porter et al. (1991)'s work.

5.4.1 Discounted cost due to deterministic technological change

From the sizing policy, at time t_n, there is a cost of installing a capacity size $U_n = X^n K_0$. The cost of installing this quantity at time 0 would be $(X^n K_0)^a$. At time t_n, with the appearance of technological change, the cost of installing this same capacity size will be discounted by a factor of $E\left[e^{-(r+p)t_n}\right]$. Then, the discounted cost of installation can be written as

$$C = E\left[e^{-(r+p)t_n}\right](X^n K_0)^a,$$

where

$$E\left[e^{-(r+p)t_n}\right] = \left(\frac{D(0)}{\gamma K_{n-1}}\right)^{\hat{\rho}},$$

and

$$\hat{\rho} = \sqrt{\left(\frac{\mu}{\sigma^2}\right)^2 + \frac{2r'}{\sigma^2}} - \frac{\mu}{\sigma^2}. \qquad (5.6)$$

Let $u(\gamma, X)$ be the infinite horizon expected capacity cost with decision variables γ and X, given by:

$$\begin{aligned}
\mu(\gamma, X) &= \sum_{n=1}^{\infty}\left(\frac{D(0)}{\gamma K_{n-1}}\right)^{\hat{\rho}}(U_n)^a \\
&= \left(\frac{D(0)}{\gamma K_0}\right)^{\hat{\rho}} \sum_{n=1}^{\infty} \frac{\left((X+1)^{n-1} X\right)^a}{(X+1)^{\hat{\rho}(n-1)}} \\
&= \left(\frac{D(0)}{K_0}\right)^{\hat{\rho}} K_0^a f(\gamma, X), \qquad (5.7)
\end{aligned}$$

where

$$f(\gamma, X) = \frac{\gamma^{-\hat{\rho}} X^{a}}{1-(X+1)^{a-\hat{\rho}}}. \tag{5.8}$$

From the assumption that $r > g$, it follows that $\hat{\rho} > 1 > a$.

5.4.2 Discounted cost due to uncertain technological change

As described in previous chapters, we assume the technological change follows a Poisson process with rate λ, the average rate of innovation per unit time. Let $N(t)$ be a number of innovations up to an arbitrary time t. Let q be the exponential rate of cost decrease for each innovation. Since $N(t)$ is a Poisson process, we have $E\left[e^{-qN(t)}\right] = e^{-\left(1-e^{-q}\right)\lambda t}$. The equivalent deterministic cost decrease rate can be given by $\hat{p} = \left(1-e^{-q}\right)\lambda$. The discount factor in deterministic technological change is now being changed to

$$E\left[e^{-(r+\hat{p})t_n}\right] = \left(\frac{D(0)}{\gamma K_{n-1}}\right)^{\hat{\beta}}, \quad \hat{r} = r + \hat{p}$$

where

$$\hat{\beta} = \sqrt{\left(\frac{\mu}{\sigma^2}\right)^2 + \frac{2\hat{r}}{\sigma^2}} - \frac{\mu}{\sigma^2} \tag{5.9}$$

Hence, the infinite horizon expected capacity expansion cost is

$$u(\gamma, X) = \sum_{n=1}^{\infty} \left(\frac{D(0)}{\gamma K_{n-1}}\right)^{\hat{\beta}} (U_n)^a,$$

$$= \left(\frac{D(0)}{\gamma K_0}\right)^{\hat{\beta}} \sum_{n=1}^{\infty} \frac{\left((X+1)^{n-1} X\right)^a}{(X+1)^{\hat{\beta}(n-1)}},$$

$$= \left(\frac{D(0)}{K_0} \right)^{\hat{\beta}} K_0^a f(\gamma, X), \tag{5.10}$$

where

$$f(\gamma, X) = \frac{\gamma^{-\hat{\beta}} X^a}{1 - (X+1)^{a-\hat{\beta}}} \tag{5.11}$$

and $\hat{\beta} > 1 > a$.

5.5 Balancing cost and shortage

The goal of the capacity expansion problem with the existence of potential capacity shortage during lead time is to minimize the total cost function comprised of discounted capacity cost and shortage penalty. The objective function can be written as

$$w(\gamma, X) = u(\gamma, X) + \hat{m}\, v(\gamma, X). \tag{5.12}$$

Here, $\hat{m}$ is a penalty factor for shortage. According to Ryan (2000), the capacity cost function $\mu(\gamma, X)$ is an increasing function of X for fixed γ, and a decreasing function of γ for fixed X. On the other hand, the shortage function $v(\gamma, X)$ increases with γ for fixed X, and decreases with X for fixed γ. Since the objective function appears to be convex in both decision variables, we can find significant optimal values of each variable for proper values of the penalty factor.

$$w(\gamma, X) = \left(\frac{D(0)}{K_0} \right)^{\hat{\rho}} K_0^a f(\gamma, X) + \hat{m} \left(\frac{D(0)}{K_0} \right)^{\rho} K_0 y(\gamma, X). \tag{5.13a}$$

For the capacity expansion problem with deterministic technological change, and

$$w(\gamma,X)=\left(\frac{D(0)}{K_0}\right)^{\hat{\beta}} K_0^a f(\gamma,X)+\hat{m}\left(\frac{D(0)}{K_0}\right)^{\rho} K_0 y(\gamma,X) \tag{5.13b}$$

for the problem with uncertain technological change.

5.6 Optimal policy parameters for capacity expansion with deterministic technological change

In this section, we apply the discounted cost due to deterministic technological change from section 5.4.1 to the objective function (5.13). Then, we use the *FindMinimum* function in *Mathematica* to calculate the optimal policy parameters γ and X. Since we have the objective function from (5.13a) as

$$w(\gamma,X)=\left(\frac{D(0)}{K_0}\right)^{\hat{\rho}} K_0^a f(\gamma,X)+\hat{m}\left(\frac{D(0)}{K_0}\right)^{\rho} K_0 y(\gamma,X)$$

where

$$f(\gamma,X)=\frac{\gamma^{-\hat{\rho}}X^a}{1-(X+1)^{a-\hat{\rho}}},$$

and

$$y(\gamma,X)=\left(\frac{S_1}{K_0}\right)\frac{\gamma^{-\rho}}{1-(X+1)^{1-\rho}},$$

the objective function (5.13a) can be restated as,

$$w(\gamma,X)=\left(\frac{D(0)}{K_0}\right)^{\hat{\rho}} K_0^a\left(f(\gamma,X)+\hat{m}\frac{K_0}{K_0^a}\left(\frac{D(0)}{K_0}\right)^{\rho-\hat{\rho}} y(\gamma,X)\right)$$

Thus, minimizing $w(\gamma,X)$ is equivalent to minimizing

$$\varpi(\gamma,X)=f(\gamma,X)+my(\gamma,X),$$

where

$$m = \hat{m}\frac{K_0}{K_0^a}\left(\frac{D(0)}{K_0}\right)^{\rho-\hat{\rho}}$$

As a baseline, we used parameter values $\mu = 0.05$ (mean logarithmic growth rate of 5% per year), $r = 0.1$ (annual interest rate), $\sigma = 0.2$ (standard deviation of logarithmic demand growth), $a = 0.7$ (economies of scale factor), and $p = 0.025$ (annual rate of cost decrease due to technological change).

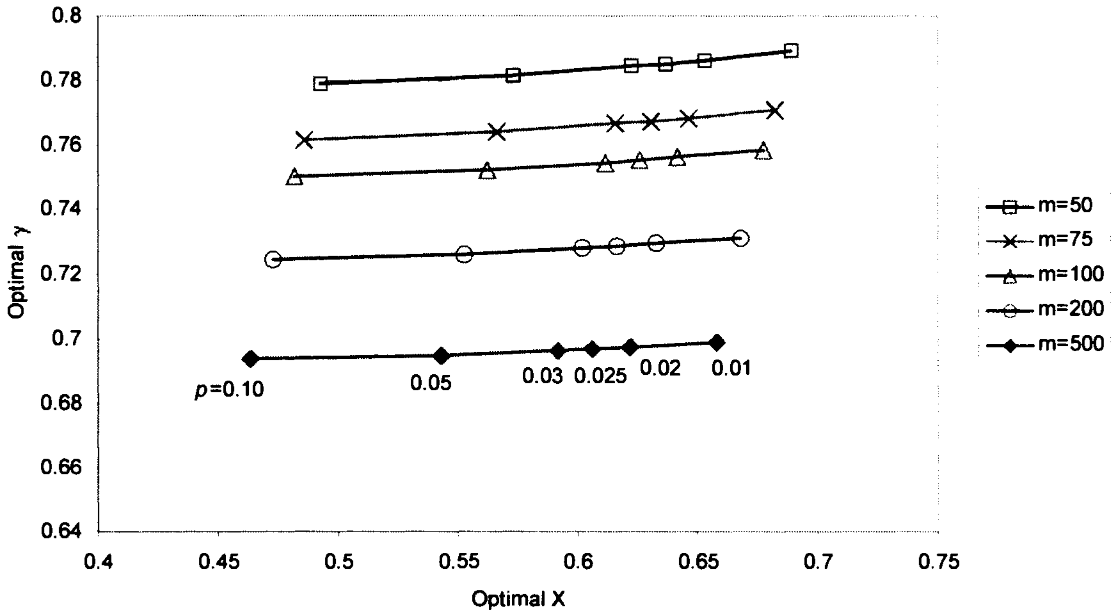

Figure 5.3 Optimal policy parameters for various annual rate of cost decrease due to technological change

Figure 5.3 illustrates the effect of technological progress on the optimal expansion policy parameters, when levels of the cost decrease rate varied from 0.01 to 0.1. As the cost decrease rate, p, becomes higher (a new model causes a large price drop of latest product model), the optimal policy tends to encourage earlier expansion and smaller size of expansion. The smaller expansion size can be explained intuitively as waiting for the price to drop before expanding, while smaller size provides for excess capacity in fewer periods of the lead time. The graphic result also shows that varying p value from 0.01 to 0.1 creates more effect to the size of expansion (X varies by 30%) than to the timing (γ varies by less than 2%). On the other hand, varying penalty factor m from 50 to 500 creates more effect to the timing (γ varies by 12%) than to the sizing (X varies by less than 6%).

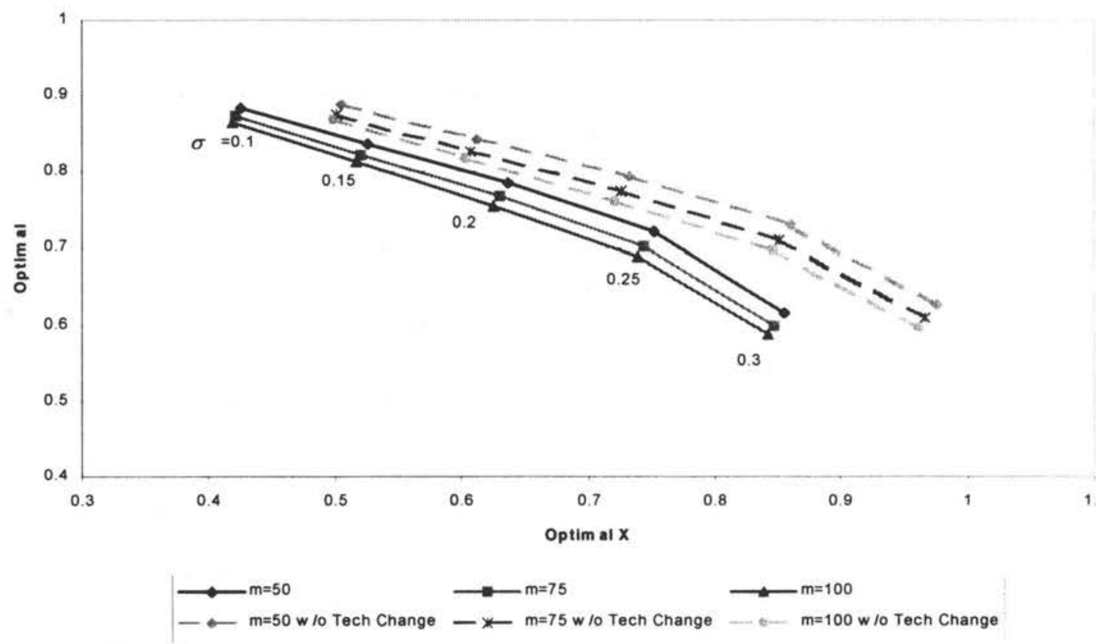

Figure 5.4 Comparing the optimal policy between expansion model with and technological change

Technological change affects the optimal policy parameter as shown on Figure 5.4. The optimal policy tends to expand slightly earlier and significantly smaller according to the numerical result. The figure also shows that when demand becomes more uncertain (larger value of σ), the optimal capacity expansions get larger and earlier.

5.7 Optimal policy for capacity expansion with uncertain technological change

In this section, we applied the discounted cost due to uncertain technological change from section 5.4.2 to the objective function (5.13b). From section 5.4.2, the discounted capacity cost is

$$u(y,X)=\left(\frac{D(0)}{K_0}\right)^{\hat{\beta}} K_0^a f(y,X),$$

where

$$f(y,X)=\frac{y^{-\hat{\beta}}X^a}{1-(X+1)^{a-\hat{\beta}}},$$

and

$$\hat{\beta}=\sqrt{\left(\frac{\mu}{\sigma^2}\right)^2+\frac{2\left(r+\left(1-e^{-q}\right)\lambda\right)}{\sigma^2}}-\frac{\mu}{\sigma^2}$$

Here, q is the exponential rate of cost decrease for each innovation, and λ is the average number of innovations per year. The objective function (5.13b) can be restated as

$$w(y,X)=\left(\frac{D(0)}{K_0}\right)^{\hat{\beta}} K_0^a\left(f(y,X)+\hat{m}\frac{K_0}{K_0^a}\left(\frac{D(0)}{K_0}\right)^{\rho-\hat{\beta}} y(y,X)\right),$$

and

$$y(\gamma, X) = \left(\frac{S_1}{K_0}\right)\frac{\gamma^{-\rho}}{1-(X+1)^{1-\rho}}$$

As a baseline, we used parameter values $\mu = 0.05$ (mean logarithmic growth rate of 5% per year), $r = 0.1$ (annual interest rate), $\sigma = 0.2$ (standard deviation of logarithmic demand growth), $a = 0.7$ (economies of scale factor), $q = 0.05$ (exponential rate of cost decrease per innovation), and $\lambda = 0.5$ (average innovation per year).

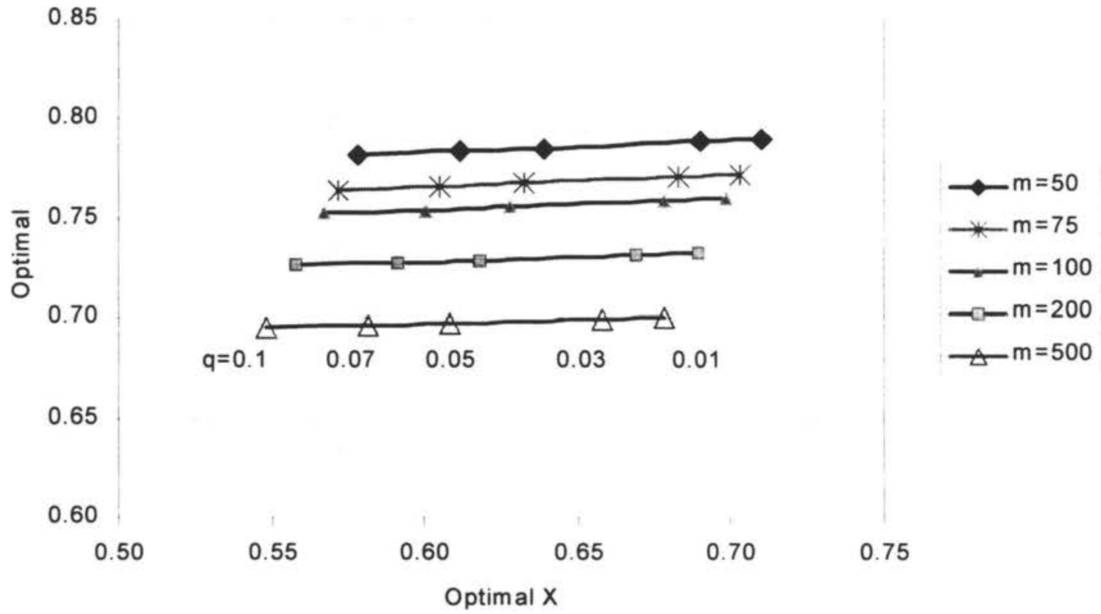

Figure 5.5 Optimal policy parameter for various q, the exponential cost decrease rate due to uncertain technological change.

For value m = 50, 75, 100, 200 and 500, Figure 5.5 show the effect on the optimal policy parameters, when rate of cost decrease per innovation varies from 0.01 to 0.1. As capacity cost has higher sensitivity to technological change (q is high), the optimal policy gets smaller and earlier. The graphic also shows that varying q has impact to sizing of expansion than timing of expansion, while varying penalty factor m has more impact to timing than expansion sizing.

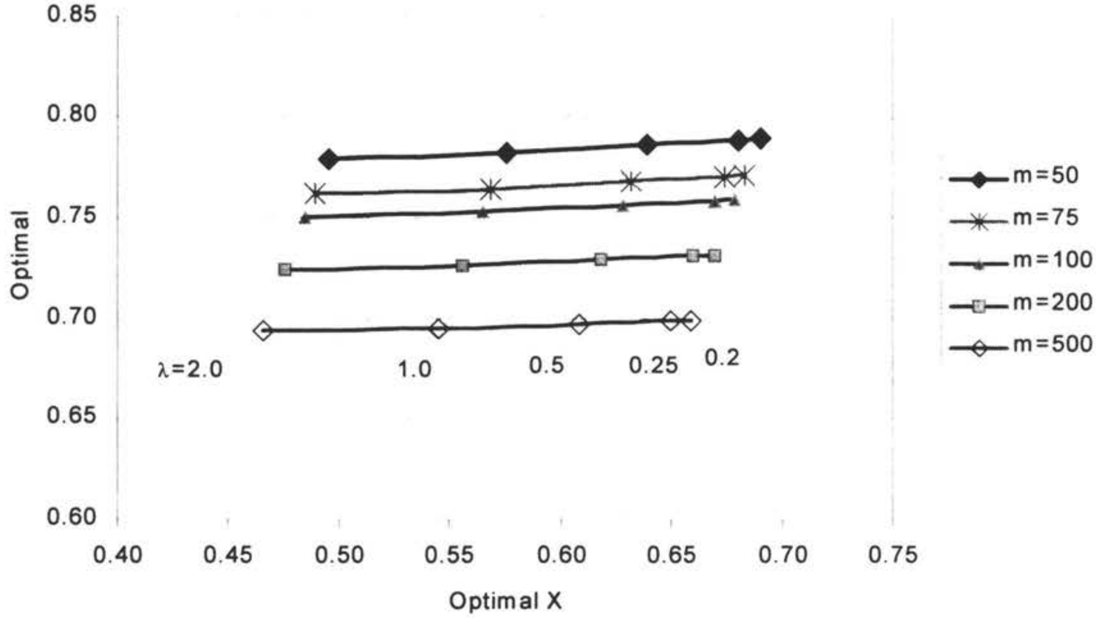

Figure 5.6 Optimal policy for various λ, Number of innovation per year.

The graphical results in Figure 5.6 show the effect of innovation rate on the optimal policy parameter, when level of annual innovation rate varied from 0.2 to 2. At high average rate (new technology occurs in the market more frequently), the optimal policy tends to encourage earlier capacity expansion earlier with smaller size. The result also shows that expansion sizing is more sensitive to λ than penalty factor. On the other hand, the timing is more sensitive to penalty factor than to λ.

CHAPTER 6. CONCLUSION AND FURTHER RESEARCH

6.1 Conclusion

The objective of this research is to study the effect of technological progress on the capacity expansion problems in several cases. Technological progress affects capacity expansion in various ways including increasing product efficiency, enlarging the market size, and reducing production cost. We assume in this research that technological progress has direct impact on decreasing the unit cost of the product, which means that it provides capacity a reduction in the present worth cost of the expanded capacity.

In Chapter 3, we investigated the capacity expansion problem with deterministic demand growth and considered both deterministic and uncertain technological change. In the deterministic case, we reviewed the capacity expansion model by Snow (1975) and solved for optimal policy parameter, which is the constant time interval between each expansion. The result shows that a larger cost decrease rate causes the optimal policy to include earlier initial expansion or a shorter time between each subsequent expansion. In the uncertain case, we modified Snow's capacity expansion model by assuming the technological progress follows a Poisson process with parameters q , the cost decrease rate per each innovation, and λ, the innovation rate per year. We used the Poisson moment generating function to identify a deterministic equivalent rate of cost decrease and used that parameter to solve for the optimal policy parameter. The calculation result shows that the qualitative effect of a high cost decrease rate is the same as the effect of a high innovation rate.

In Chapter 4, we investigated a more complicated capacity expansion model with random demand growth combined with both deterministic and uncertain technological change. We applied the deterministic equivalent interest rate suggested in Bean et al.'s research (1992) to our models in this chapter. Again, the result shows that technological progress causes the optimal policy to expand more frequently or shorter time between each expansion, which causes smaller size of expansion and more flexibility to utilize newer technology.

In Chapter 5, we incorporated the technological progress into the capacity expansion models with lead time of construction. We modified the previous works by Ryan (2000) and Pak (2001) with both deterministic and uncertain technological progress. We used the Summing European option value in order to estimate the capacity shortage during the lead time of expansion. With the addition of lead time, the optimal policy consists of both a timing and a sizing policy. The result from calculation shows that the optimal timing parameter is more sensitive to the penalty factor for capacity shortage than the technological parameters. On the other hand, the optimal sizing parameter is more sensitive to technological parameters than the shortage penalty.

6.2 Further research

The consideration of capacity expansion models with different demand growth such as linear or a step function is one interesting topic for further research. In the random demand case, the transformed Brownian motion model will be different.

The impact of technological progress on the demand growth is another interesting topic. In this thesis, we assume that they are independent. However, in some technology driven markets, the introduction of new innovations can affect the growth rate of demand. The relation between this technology rate and demand growth rate will make the problem more complicated.

An interesting extension to be considered is modeling the lead time as a controllable variable or as a random factor. In addition, the consideration of the lead time as a function of capacity size is interesting for some practical industries such as power or energy generation.

Another possible extension is some other models for technological change and their impact on the cost of expansion. We assume in this thesis that technological change decreases costs exponentially, which might not exactly fit in some capacity expansion problems.

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